

Phys. Lab. Exp.

- 1 ✓
- 2 ✓
- 4 ✓
- 5 ✓
- 6 ✓
- 8 ✓
- 11 ✓
- 12 ✓
- 16 ✓
- 18 ✓
- 20 ✓
- 25 ✓
- 30 ✓

~~_____~~

0.5 cm.

~~_____~~ mm.

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.
- 7.
- 8.
- 9.
- 10.
- 11.
- 12.
- 13.

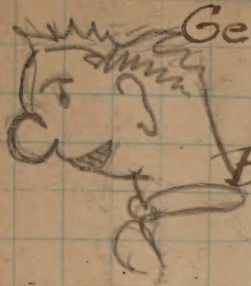
- ~~3~~
- 9 ✓
- 13 ✓
- 14 ✓
- 15 ✓
- 17 ✓
- 22 ✓
- 23 ✓
- 24 ✓
- 29 ✓
- Special.

4.75 cm.

4750 mm.

- 10 ✓
- 26 ✓
- 27 ✓
- 3
- 7

14 dw.
diff.



George W. Rocklein
'29

Brooklyn Polytech.

VE

~~1.98~~

6

1

6.196 mm. 0.002 cm.

619 cm.

6

0.02 mm.

0.05

0.048 cm.

$\frac{1}{50}$ mm.

0.002 mm.

98

1.96 mm

5 mm.

619 cm.

9.184

5390

4102

1288

1.514

1.513

1.513

1.510

1.510

1.516

1.517

1.517

1.513

9185

539

3795

4.102

1975

2

5.3950 = .539 mm.

92

92

184

920

51

102

4.102

12

13.761

1.513

920

9198

920

918

916

PHYSICAL
LABORATORY EXPERIMENTS
FOR
ENGINEERING STUDENTS

BY

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40 ILLUSTRATIONS

PART I.

MECHANICS, SOUND, HEAT, AND LIGHT

SECOND EDITION, CORRECTED



NEW YORK
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PREFACE

THE material in this volume was prepared for the use of sophomore students in the Polytechnic Institute of Brooklyn. All of these students are candidates for engineering degrees and most of them, when they entered as freshmen, had pursued courses in laboratory physics in the high schools of Greater New York, which are generally supplied with superior laboratory equipments. The book comprises thirty exercises, and the performance of each is designed to occupy three hours of the student's time. The course, therefore, covers three hours per week for two semesters. Each experiment has been chosen because of its close connection with engineering work, and in many cases the theoretical result may be calculated from the constants of the apparatus with which that result obtained by experiment may readily be compared. As these two results approach to an equality the student gains confidence in the apparatus, confidence in the theory, and confidence in himself. Apparatus of engineering design has been chosen for each exercise so that the student may rely upon getting the same result under the same conditions, and the results obtained by different students are directly comparable with each other. Each exercise is self-contained and assumes a knowledge on the part of the student of the subject matter as treated in courses on college physics. *College Physics* by Reed and Guthe is the text used at the Polytechnic. The additional theory necessary for a thorough understanding of an exercise is suggestively given with a view to inspiring the student to think and to supply the gaps.

It is believed that many other teachers of laboratory physics to engineering students may be confronted with the same conditions as exist at the Polytechnic. It is hoped that this text may be of some service to them in the solution of their problems:

S. S.
E. H.

POLYTECHNIC INSTITUTE OF BROOKLYN,
January, 1917.

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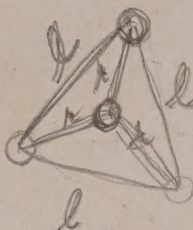
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200

.6.241

$$\frac{46}{200} = .623$$

624



PHYSICAL LABORATORY EXPERIMENTS

EXPERIMENT 1

Radius of Curvature of Lenses by Spherometer

OBJECT. To determine the radius of curvature of a concave and of a convex lens with the use of a spherometer, a micrometer caliper, and a vernier caliper.

THEORY. The spherometer consists of a screw moving vertically in a nut mounted at the center of an equilateral tripod. The instrument is arranged to measure accurately the distance between the point of the screw and the plane of the three tripod legs. If the spherometer reading be a and the distance between the axes of the screw and tripod leg be r , it follows that the radius of a vertical circle which passes through the point of the screw and the end of one tripod leg, and whose center lies on the prolonged axis of the spherometer screw, is

$$R = \frac{r^2}{2a} + \frac{a}{2}.$$

The distance r can be expressed in terms of the mean length l between centers of the tripod legs, from the geometry of the equilateral triangle formed by these legs. Thus

$$r = \frac{l}{\sqrt{3}},$$

and substituting this value in the foregoing equation, then

$$R = \frac{l^2}{6a} + \frac{a}{2}$$

2.174

APPARATUS. Spherometer, micrometer caliper, vernier caliper, plane surface, concave and convex lenses.

Spherometer. The spherometer, Fig. 1, carries a graduated circular disk at the top of the screw, and a fixed vertical scale graduated in divisions equal to the pitch of the spherometer screw. The whole number of revolutions of the screw can be read on the vertical scale and the fractions of a turn can be read on the disk. To take a zero reading, place the instrument on a true plane surface and turn the screw slowly by means of the knurled head until the entire spherometer just begins to revolve around the screw point as pivot. When this occurs without noticeable rocking of the instrument on the surface,

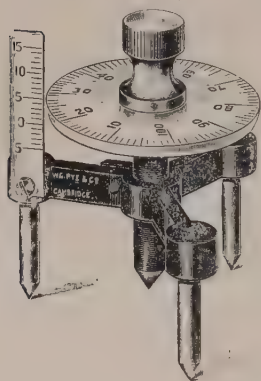


FIG. 1.

the reading is taken by observing the indications on the vertical and circular scales and adding them. The settings of the circular scale should be estimated to tenths of divisions. Readings on other surfaces are made similarly.

Micrometer caliper. The micrometer caliper, Fig. 2, is based upon the same operating principle as the spherometer. It consists of a screw moving in a U-shaped frame, the screw being provided with a hollow graduated head which can be turned back and forth over a fixed longitudinal scale. An object to be measured is placed between the screw and the anvil and the screw is turned until a slight pressure is felt. The whole turns of the

screw are read upon the fixed scale and the fractions of a turn are read upon the micrometer head; together they indicate the thickness of the object. Calipers are fre-

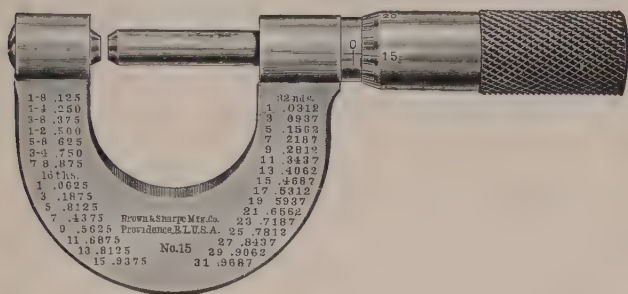


FIG. 2.

quently provided with a ratchet head for guarding against the application of excessive pressure on the micrometer screw.

Vernier caliper. The vernier caliper, Fig. 3, consists

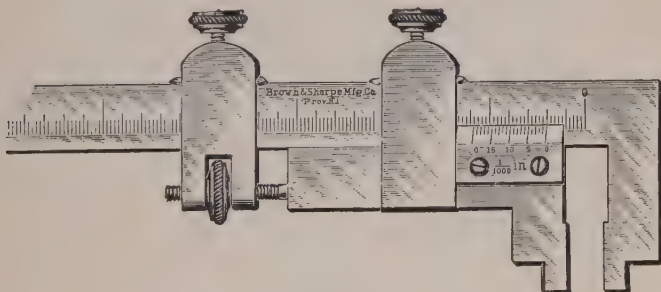


FIG. 3.

of a slider with a projecting jaw that can be moved along a graduated bar or scale also provided with a jaw, the instrument being designed to measure accurately the distance between these jaws by means of the scale and by the vernier carried on the slider. The vernier has n equal

spaces or divisions whose aggregate length is equal to that of $n \pm 1$ of the smallest divisions on the scale. Therefore the length of each vernier division is $\frac{n \pm 1}{n}$, or $1 \pm \frac{1}{n}$, of the smallest scale division, that is, the vernier division is $\frac{1}{n}$ of the smallest scale division more or less than such division. This slight difference in the lengths of vernier and scale divisions, called the least count L of the vernier, enables the determination of fractional parts of divisions on the fixed scale with precision. In measuring the length of an object, the jaws of the caliper are brought into light contact with the object, and the scale reading up to the zero point or index of the vernier is observed. When the index does not exactly meet a mark on the scale, the length of the fractional scale division is determined by observing which vernier mark coincides exactly with a mark on the scale. If this coincidence occur at the q th vernier mark, the length of the fractional scale division is evidently qL , and this amount is to be added to the direct scale reading.

PROCEDURE. Take four readings with the spherometer on each lens and on the plane surface. Measure the diameter of each of the cylindrical spherometer legs four times with the micrometer caliper. Measure with the vernier caliper the distances between the outsides of every two spherometer legs four times. From these measurements and those made with the micrometer caliper, compute the average distance between the contact points of the spherometer legs. Make zero corrections in all instrument measurements. In making spherometer measurements on a convex lens after having taken the zero reading on plane glass observe that the spherometer screw is turned in a direction opposite to that of the circular scale.

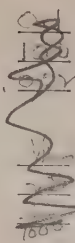
OBSERVATIONS. The observations should be recorded in tabular form as follows:

*Instrument Constants***Spherometer:**

Length of divisions on fixed scale

Number of divisions on disk

Value of each disk division

 cm.

cm.

Micrometer caliper:

Length of divisions on fixed scale

Number of divisions on micrometer head

Value of each division on head

in.

in.

Vernier caliper:

Length of smallest scale division

Number of divisions on vernier

Least count of vernier

in., cm.

in., cm.

in., cm.

Spherometer Readings (cm.)

On plane glass

On concave lens

On convex lens

Aver. —

Aver. —

Aver. —

(a) = —

(a) = —

Diameter of Tripod Legs (in.)

Zero reading

Leg A

Leg B

Leg C

Aver. —

Aver. reading on the three legs

Mean diameter of tripod leg $\frac{1514}{1000}$ in. = $\frac{383}{1000}$ cm.

Distance between Outsides of Tripod Legs (cm.)

Zero reading	Leg A to Leg B	Leg B to Leg C	Leg A to Leg C	
<u>0</u>	4.77	4.74	4.73	4.77
<u>0</u>	4.79	4.76	4.74	4.76
<u>0</u>	4.78	4.77	4.74	4.76
<u>0</u>	4.76	4.77	4.75	4.76
Aver. <u>0</u>	Average distance 4.74			
Mean distance between outsides of tripod legs				4.78 cm.
Mean distance between centers of tripod legs (l)				4.39 cm.



CONCLUSIONS. Calculate the radii of curvature of the lenses from the foregoing equations.

$$R = 11.4 \text{ cm.}$$

$$\text{new } R = 14.73 \text{ cm.}$$

EXPERIMENT 2**Measurement of Areas by Planimeter**

OBJECT. To determine the areas of several irregular plane figures by means of the Amsler planimeter. The figures to be measured are: a current-time curve, a hysteresis loop, a railway speed-time curve, and an indicator diagram; these are given on the accompanying detachable sheet.

THEORY. The planimeter, Fig. 4, is an instrument consisting essentially of a beam which carries a tracing point or stylus at one end and a graduated wheel with a smooth protruding rim near the other end, the axis of the wheel being parallel to the beam. An auxiliary arm is hinged to the beam at some convenient point, while the other end of this arm carries a needle point. The instrument, when placed upon a horizontal surface, rests upon the needle point, the stylus and the periphery of the wheel. To measure the area of a closed plane figure drawn on smooth paper, the stylus is carefully moved once around the perimeter of the figure, while the needle

point maintains a fixed position by being pressed into the paper. During this movement the wheel rotates and slides; the rotatory motion results from a movement of the beam in a direction perpendicular to itself, and the sliding motion results from a movement of the beam along its own direction. The net rotation of the wheel for one complete transit of the stylus along the perimeter of a figure indicates the area of that figure directly, when the needle point is fixed outside of that area.

In Fig. 5, let AB be the planimeter beam of length l , AN be the auxiliary arm with its needle point fixed at N ,

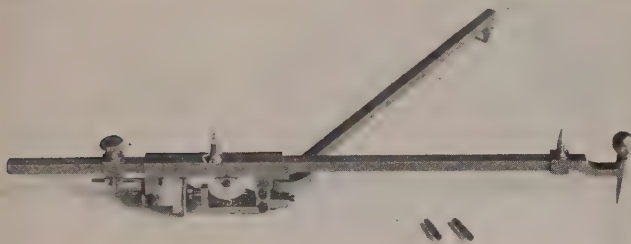


FIG. 4.

and W be the position of the wheel which is located a distance d from the center point of AB . Consider the motion of the beam while the stylus B moves a very small distance BB''' along the perimeter of the figure whose area S is to be measured, this distance being greatly exaggerated in the figure for the sake of clearness. The movement of the beam from AB to $A'''B'''$ may be resolved into three simple component movements, as follows: 1, a motion of rotation about W to the position $A'B'$; 2, a motion of translation perpendicularly to itself to the position $A''B''$; and 3, a motion of translation along itself to the position $A'''B'''$, the auxiliary arm being considered detached during these movements.

Areas are swept out by the beam during the first and second of these component movements, and when such areas are swept out toward the right they are considered positive, and toward the left negative. The net area of rotation of the beam through the angle θ is equal to the area of the sector $BB'W$ minus the area of the sector $AA'W$, which can be shown to be equal to θld . The distance s through which the beam moves perpendicularly to itself is best measured by noting the angular movement of the wheel in moving from W to w . The diagram in

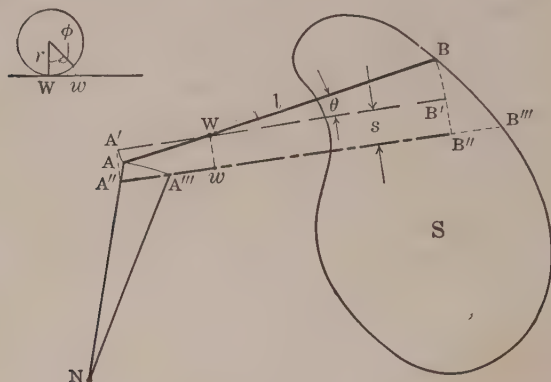


FIG. 5.

the upper corner of Fig. 5 shows that the wheel of radius r turns through the angle ϕ in order for the wheel to move from W to w ; therefore $s = r\phi$. The area of the rectangle $A'A''B''B'$ which is swept out by translation is equal to $lr\phi$. Consequently the total area swept out by the beam when the stylus traverses the elementary portion BB''' of the perimeter of the figure is $\theta ld + lr\phi$. When the stylus moves once completely around the perimeter of the closed figure in a clockwise direction, any area outside of the figure which is swept over by the beam is swept out once toward the right and once toward the left, while the area of the figure itself is

swept out only toward the right, therefore the area of the figure is

$$S = \Sigma \theta l d + \Sigma l r \phi.$$

Since the beam resumes its original position after a complete transit of the stylus, $\Sigma \theta = 0$, whence

$$S = l r \Sigma \phi,$$

where $\Sigma \phi$ is the net rotation of the wheel during one complete transit of the perimeter by the stylus. It is evident from this equation that the periphery of the wheel of a planimeter may be graduated to read directly in units of area.

When the area that is to be measured is very large, it is convenient to place the needle point inside of the area. The planimeter reading taken after one complete transit of the perimeter is less than the area of the figure. The true area is obtained by adding to the planimeter reading (with attention to sign) the area of a datum circle which is generated by the stylus when it moves about the needle point as center so that the wheel will not rotate. (For proof, see p. 59 of Ferry and Jones, "Practical Physics").

APPARATUS. Planimeters of different makes, drawing boards, triangle, micrometer caliper, and scale.

PROCEDURE. Measure with the scale the length of one planimeter beam, l , from the stylus to the point of attachment of the auxiliary arm. In some instruments, Fig. 4, this distance may be readily measured from point to point of the upwardly projecting pins on the beam. Measure the diameter, $2r$, of the wheel over its rim with the micrometer caliper. Show that the area represented by one revolution of the wheel is $2\pi l r$.

Draw a rectangle having an area of from 5 to 10 square inches, and measure its area by a planimeter four times and compare the average value with the product of the two dimensions of the rectangle as measured with the scale.

To use the planimeter, place the needle point at a

convenient spot outside of the area to be measured. Hold the stylus at any convenient starting point, set the graduated wheel to its zero position against the index of the vernier, and then trace the perimeter of the figure in a clockwise direction with the stylus until the starting point is again reached. Read the new position of the wheel with the aid of the vernier. Take four readings on each figure with one planimeter.

Measure the areas of the current-time curve, hysteresis loop, speed-time curve, and indicator diagram, each four times, in square inches, with one of the planimeters. If the beam of the planimeter is of adjustable length, set it to read square inches.

OBSERVATIONS. Record observations as below:

Planimeter Constants

Name of Make:	Ashcroft, Dietzgen, or Crosby
Length of beam	(l) = <u>14</u> in.
Diameter of wheel	($2r$) = <u>1</u> in.
Area per revolution of wheel	$2\pi lr$ = <u>1.57</u> sq. in.
Highest reading on wheel	— sq. in.
Percentage difference	—
Value of smallest wheel division	— sq. in.
Least count of vernier	— sq. in.

Readings on Rectangle

Planimeter readings (sq. in.)	Length of rectangle <u>1.5</u> in.
<u>1.5</u>	Height of rectangle <u>1.5</u> in.

Aver. — Calculated area 2.25 sq. in.

Percentage difference 2.25%

Area Measurements on Irregular Figures (sq. in.)

Current curve	Hysteresis loop	Speed-time curve	Indicator diagram
<u>1.5</u>	<u>1.5</u>	<u>1.5</u>	<u>1.5</u>
<u>1.5</u>	<u>1.5</u>	<u>1.5</u>	<u>1.5</u>
<u>1.5</u>	<u>1.5</u>	<u>1.5</u>	<u>1.5</u>
<u>1.5</u>	<u>1.5</u>	<u>1.5</u>	<u>1.5</u>
Aver. —	<u>1.5</u>	<u>1.5</u>	<u>1.5</u>

CONCLUSIONS. Reduce the areas of the irregular figures to their proper units as indicated by their ordinates and abscissas, and determine their average heights. Explain the method of reduction and record results as follows:

Figure	Area	Average height
Current curve	1.8 coulombs	61.85 amperes
Hysteresis loop	gilbert-max-wells per cu. cm.	
Speed-time curve	48 miles	72.50 miles per hr.
Indicator diagram		lbs. per sq. in.

EXPERIMENT 3

Acceleration of Gravity by Atwood's Machine

OBJECT. To verify the laws of uniformly accelerated motion and to determine the acceleration of gravity by means of Atwood's machine.

THEORY. Atwood's machine is used to measure the velocity and acceleration of bodies by observing the spaces that are traversed by them during known intervals of time. For convenience, two bodies of equal mass are affixed to the ends of a thin cord which is hung over a light freely moving wheel, and the acceleration is produced by the force of gravity acting on a supplementary mass added to one of the bodies. In verifying the laws of uniformly accelerated motion, the expressions used are

$$s = \frac{1}{2}at^2,$$

and

$$v = \sqrt{2as},$$

where s is the distance traveled from rest in t seconds by a body moving with uniform acceleration a , and v is the velocity of the body acquired after moving a distance s .

If a small weight of mass m be placed upon one of the

two equal bodies, the force of gravity imparts to the total mass an acceleration

$$a = \frac{mg}{M + m},$$

where g is the acceleration of gravity and M is the aggregate mass of the two bodies. By eliminating a from the foregoing equations, an expression for the acceleration of gravity may be obtained. Inasmuch as the wheel itself has also been accelerated, its equivalent mass should be added to $M + m$. This equivalent mass is equal to its moment of inertia I divided by the square of the distance r from its center to the cord. Introducing this factor, the final equation for the acceleration of gravity becomes

$$g = \frac{2s}{mt^2} \left(M + m + \frac{I}{r^2} \right).$$

APPARATUS. Atwood's machine, metronome, stop-watch, balance with weights, and meter-stick.

The Atwood's machine comprises a light pulley supported at the top of a frame, two cylindrical bodies of equal mass attached to the ends of a cord which passes over the wheel, a metal rider to serve as the accelerating weight, a platform for arresting the motion of the descending body, and a ring stage for removing the rider from the body at particular instants during its descent.

To take an observation, level the instrument, place the cylindrical bodies with the attached cord in position, put the rider or other weights on the rear body, and draw the front body to its lowest position. Start the metronome and exactly on a click of this device release the front body. The platform should then be raised or lowered after each trial until in its final adjustment the descending rear body will strike it simultaneously with a metronome click. When the accelerating force is to act only during a part of the descent, the ring stage is first adjusted to remove the rider exactly on a metronome click, and then

the platform is adjusted at a lower position to stop the motion exactly on another click. The moment of inertia of the wheel is 150 gram-cm.^2 , and its radius is 5.1 cm.

PROCEDURE. Determine the weight in grams of the rider and of the two bodies with string. Set the metronome weight so that its upper edge indicates 120, and then count the number of clicks made in two minutes. From this count derive the time interval in seconds between two successive clicks.

To verify the laws of accelerated motion. (a) With the rider on the rear body, measure with a meter-stick the spaces through which it moves during 2, 3 and 4 intervals between metronome clicks and show that these spaces are to each other as are the squares of the respective times of descent. (b) Allow the rider to act only for 2 intervals between clicks and measure the distance traversed subsequently during 2, 3 and 4 such intervals to show that the descending body travels with constant velocity after removal of rider. (c) Allow the rider to act for 1, 2 and 3 intervals between clicks and measure in each case the distance traversed during two such intervals after removal of the rider, in order to show that velocities acquired by the descending body while accelerating are directly proportional to the times of descent. Measure all distances to the nearest millimeter.

To determine the acceleration of gravity. Place weights of 10, 20, 30, 40 and 50 grams successively on the rear body and measure the distances it traverses while descending to the platform in some integral number of intervals between metronome clicks, selecting that number which gives the largest descent on the machine. Verify the position of the platform by repeated trials for each added weight.

OBSERVATIONS AND CONCLUSIONS. Record all observations and arrange them in tabular form, and verify the laws of uniformly accelerated motion as already indicated. In the determination of the acceleration of gravity calculate the acceleration of the descending body by means

of $a = 2s/t^2$, and plot a curve with a as ordinates against added masses on descending body as abscissas. The points will lie approximately in a straight line, and such a line is to be drawn which will pass as nearly as possible through all of the observational points. The intercept of this line on the axis of abscissas indicates that part of the added masses which is required to overcome friction in the wheel. In the calculations this intercept should be subtracted from the added masses in order to obtain m . Calculate for each value of m , the acceleration of gravity in cm. per sec. per sec., and compare the average value for g as obtained by test with the theoretical value as given in Tables II and IX.

EXPERIMENT 4

Acceleration of Gravity by Falling Body

OBJECT. To determine the acceleration of gravity by means of a body falling freely under the force of gravity before a vibrating tuning fork.

THEORY. A stylus on a vibrating tuning fork of known frequency f is adjusted to trace a sinuous line on the falling body. The distances between crests of this line will be found to increase as the body descends because of its acceleration. Mark a transverse line near the beginning of the trace where the curves are perfectly formed, and from this line mark off spaces by similar transverse lines, each space containing exactly 10 wave-lengths. Call these spaces s_1, s_2 , etc. Since the body had acquired some velocity v_0 upon reaching the first mark, it follows that the successive spaces described in the time $\frac{10}{f} = t$ seconds, which corresponds to 10 vibrations of the fork, are

$$\begin{aligned}s_1 &= v_0 t + \frac{1}{2} g t^2, \\ s_2 &= v_0 t + \frac{3}{2} g t^2, \\ s_3 &= v_0 t + \frac{5}{2} g t^2, \text{ etc.,}\end{aligned}$$

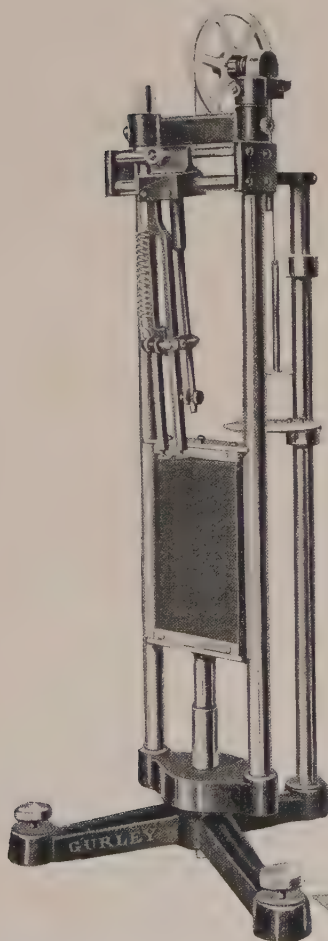


FIG. 6.

where g is the acceleration of gravity. Then, if S cm. be the mean space increment, that is, the average value of $s_2 - s_1$, $s_3 - s_2$, etc., the acceleration of gravity in cm. per sec. per sec. will be

$$g = \frac{S}{t^2}.$$

APPARATUS. Falling-plate device with electrically operated tuning fork, glass or metal plate, battery, scale and scribe.

The device employed in this measurement consists of a vertical metal frame in which slides a metal carrier holding a piece of plate glass, Fig. 6. The glass plate is painted with a thin paste of whiting mixed with alcohol, and when dry is clamped to the carrier by means of two spring hooks. This carrier is raised and should be fastened at the top of the frame by a string which is passed through the hole in the cross-bar and fastened by being wrapped around a screwhead in the cross-bar. On the frame is mounted an electrically operated tuning fork which has a stylus on one prong. To take an observation, close the electrical circuit of the tuning fork and start the fork vibrating, adjusting the contact screw if necessary. Turn the milled screw on the tuning-fork holder until the stylus bears properly upon the glass, and then burn the string by applying a lighted match above the cross-bar. The descending plate will receive a fine tracing on its whitened surface.

PROCEDURE. Level the instrument so that the carrier may fall freely with the least amount of friction, and take an observation as outlined. Displace the fork about $\frac{1}{2}$ inch in a horizontal direction and take another observation. Several trials may be necessary to yield good results. When two good traces have been obtained, remove the glass plate and rule a transverse mark upon every tenth crest of each trace with a scribe and scale. Measure the distance between successive transverse marks to hundredths of an inch.

OBSERVATIONS. Denote the distances between the transverse marks by s_1, s_2 , etc., beginning at the end of the trace where the wave-lengths are short, and record the measurements as below:

0.03509

Trace Measurements (in.) 285

Distances traversed in $\frac{10}{f} = t$ sec.		Differences between successive distances	
Trace 1	Trace 2	Trace 1	Trace 2
$s_1 = \underline{.52}$	$\underline{.116}$	$s_2 - s_1 = \underline{.65}$	$\underline{.55}$
$s_2 = \underline{1.17}$	$\underline{1.71}$	$s_3 - s_2 = \underline{.40}$	$\underline{.42}$
$s_3 = \underline{1.57}$	$\underline{2.13}$	$s_4 - s_3 = \underline{.38}$	$\underline{.47}$
$s_4 = \underline{1.95}$	$\underline{2.60}$	$s_5 - s_4 = \underline{.51}$	—
$s_5 = \underline{2.46}$	—		
Mean space increment		$S = \underline{.425}$ in. = $\underline{1.065}$ cm.	

CONCLUSIONS. The differences between the successive distances s_1, s_2 , etc., should all be equal, since the acceleration of gravity is constant. The rate of vibration of the tuning fork is 285 per second, therefore the time interval $t = \frac{10}{285} = 0.03509$ second. Compute the value of the acceleration of gravity in cm. per sec. per sec. and compare this value with the theoretical value at the place where the experiment is performed, see Tables II and IX.

EXPERIMENT 5

Coefficient of Restitution and Hardness by Scleroscope

OBJECT. To determine the coefficient of restitution and the relative hardness of metals by means of a Shore scleroscope.

THEORY. That number by which the relative velocity of two colliding bodies just before impact must be multiplied in order to give their relative velocity just after impact is called the coefficient of restitution of the bodies. If one of the bodies be clamped stationary, then the coefficient of restitution is the ratio of the velocities of the

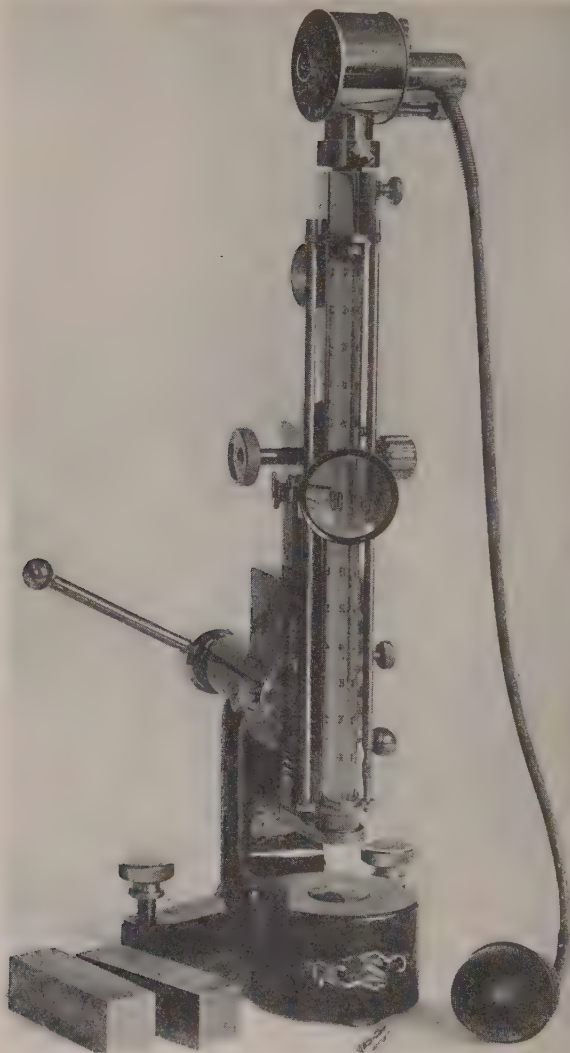


FIG. 7.

other body just after and just before impact. In the scleroscope, a miniature diamond-pointed hammer is allowed to fall freely from a known constant height h_1 upon the specimen under test, which is held stationary. If the velocity of the hammer at the instant of striking the specimen be v_1 , its velocity just after impact be v_2 , and the height to which the hammer rebounds be h_2 , then it follows from the law of falling bodies that the coefficient of restitution is

$$k = \frac{v_2}{v_1} = \sqrt{\frac{h_2}{h_1}}.$$

$$\frac{d_1 = \frac{v_1^2}{2g}}{\frac{d_2 = \frac{v_2^2}{2g}}{\frac{v_2^2}{v_1^2} = \frac{d_2}{d_1} = k^2}}$$

The rebound h_2 can serve directly as a measure of the hardness of a specimen on an arbitrary scale. The scale on the Shore scleroscope has 140 equal divisions, and on this scale hard steel corresponds to a rebound of 100. Then, if the hammer rebounds from a test piece to division 55 on this scale, the piece is said to be "55 hard." It is interesting to note that the ultimate tensile strength of a material is quite closely related to its hardness as measured by the scleroscope.

In the instrument used in this experiment the hammer falls from a height of 9.90 inches, and each division of the scleroscope scale measures 0.0643 inch; therefore

$$k = 0.0806\sqrt{d},$$

where d is the hardness or the number of divisions rebound.

APPARATUS. Scleroscope with universal hammer, magnifier hammer, five steel specimens of the same carbon content but quenched at different temperatures, and soft metal specimens of copper, brass and bronze.

The principal feature of the scleroscope is a small diamond-pointed hammer of about 40 grains mass which falls within a glass tube onto the metal specimens clamped at the base, Fig. 7. The hammer is raised to the top of the instrument by suction and it is released by an ingenious pneumatically operated mechanism, shown in

Fig. 8. A hand bulb connected to tube *A* acts on timer cam *B* through the medium of the piston *C*, the adjustable oscillator *D* and the end ratchets *E*. When the bulb is pressed the cam acts on the valve *F*, unseating it, and shortly thereafter the hammer *G* is released from the hook prongs *H* because of the downward movement of the hollow cone *I*. During the descent of the hammer

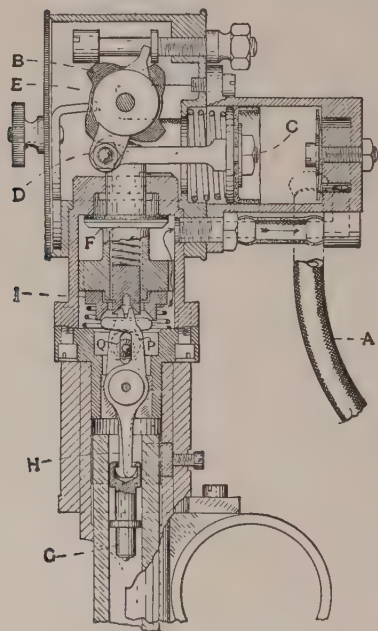


FIG. 8.

there will be free access of air to the glass tube. When the bulb is again pressed the cam seats valve *F*, and the tube chamber is placed in direct communication with the bulb, so that when the bulb is subsequently released the reduction in pressure in the tube causes the hammer to be drawn up and caught on the hook prongs.

Before taking an observation, turn the leveling screws

in the base until the instrument is level, as indicated by the bob-rod alongside of the tube barrel. Press the bulb and then release it quickly so as to raise the hammer to its clutch at the top of the tube. To take an observation on a flat specimen, raise the clamping shoe by the lever at the rear, insert the specimen, bring down the clamping shoe, and hold it firmly in this position while making the test. Press the bulb to release the hammer and note quickly its height of rebound, the top of the hammer serving as the indicator. A preliminary reading viewed at a little distance will enable one to properly locate the magnifying glass on the scale for the final readings. Never allow the hammer to drop without striking some clamped object under it, for otherwise it is likely to break the glass. For greater accuracy in measuring the hardness of soft metals a "magnifier hammer" replaces the diamond-pointed hammer.

PROCEDURE. The five samples of steel to be tested have the same percentage of carbon content, but were subjected to different heat treatments. The temperatures in degrees Fahrenheit at which these samples were quenched in the process of hardening are stamped on their ends. Take two readings at different spots on each steel specimen.

Remove the standard or universal hammer under the direction of the laboratory instructor and substitute the magnifier hammer, which gives rebounds 1.78 times as great as the universal hammer for otherwise identical conditions. Take two readings on copper,³ on brass, and on phosphor bronze with the magnifier hammer and reduce these readings to the standard scale.

OBSERVATIONS AND CONCLUSIONS. Arrange data in tabular form and determine the hardness and coefficient of restitution of each sample tested. Compare the hardness values obtained by test with the values given in Table III. For the steel samples plot curves of hardness and of coefficients of restitution as ordinates against quenching temperatures as abscissas.

EXPERIMENT 6

Moment of Inertia of Rotating Wheel

OBJECT. To determine the moment of inertia of a cylindrical wheel by measuring its acceleration when subject to a known torque and by calculation from its mass and dimensions.

THEORY. Consider an annular element of radius r and of width dr , within a cylinder of outside radius R , of axial length l , and of density δ . The mass of this element is

$$dm = 2\pi r l \delta dr.$$

If a torque $d\mathcal{T}$ acts on this element, its particles will acquire the linear acceleration a and the angular acceleration α ; whence

$$d\mathcal{T} = ar dm = \alpha r^2 dm = 2\pi l \delta \alpha r^3 dr.$$

By integration, the total torque acting on the entire cylinder becomes

$$\mathcal{T} = \frac{\pi}{2} l \delta \alpha R^4 = \frac{1}{2} M R^2 \alpha,$$

since the total mass of the cylinder $M = \pi R^2 l \delta$. The portion $\frac{1}{2} M R^2$ is a constant of the cylinder called its moment of inertia, and if this constant be represented by I , the expression for the constant torque becomes

$$\mathcal{T} = I \alpha.$$

The angular acceleration is measured as in Experiment 4 by allowing a stylus on a vibrating tuning fork of known frequency to trace a sinuous line on the periphery of the wheel, and by measuring the angular distances moved through in equal intervals of time. If the angles sub-

tended by successive groups, each of 50 wave-lengths, be denoted by $\theta_1, \theta_2, \theta_3$, etc., then

$$\begin{aligned}\theta_1 &= \omega_0 t + \frac{1}{2} \alpha t^2, \\ \theta_2 &= \omega_0 t + \frac{3}{2} \alpha t^2, \\ \theta_3 &= \omega_0 t + \frac{5}{2} \alpha t^2, \text{ etc.},\end{aligned}$$

where ω_0 is the angular velocity of the wheel upon reaching the beginning of the first group of waves, and t is the common time corresponding to 50 complete vibrations of the tuning fork. From these equations it is evident that the angular acceleration α is the average of

$$\alpha = .844 \frac{\theta_2 - \theta_1}{t^2}, \frac{\theta_3 - \theta_2}{t^2}, \text{ etc.}$$

APPARATUS. Heavy pivoted wheel with electrically operated tuning fork, battery, accelerating weight, friction-balance weight, try-square, scale, caliper, and scriber.

The wheel is mounted as shown in Fig. 9 so as to revolve about a horizontal axis. The constant torque to be applied in order to revolve the wheel with uniformly accelerated motion is obtained by a weight attached to a cord which passes around the small cylindrical projection on the side of the wheel. The tuning fork is mounted on a carriage which can be moved in a direction parallel to the axis of the wheel, so that the stylus on the fork may trace out a sinuous line extending over several revolutions of the wheel without overlapping. One side of the wheel is graduated in degrees.

PROCEDURE. Coat the rim of the wheel with a mixture of whiting and alcohol, attach one end of a string to the projection on the wheel with a small amount of wax and place the friction-balance weight and an accelerating weight of 200 grams on the other end of the string. Wind up the string and close the circuit of the tuning-fork magnet. Adjust the stylus by means of the knurled

screw on the carriage until it bears properly upon the wheel and then release the wheel. As the wheel revolves, slowly move the fork by means of the lever at its base so that the stylus travels axially across the whitened

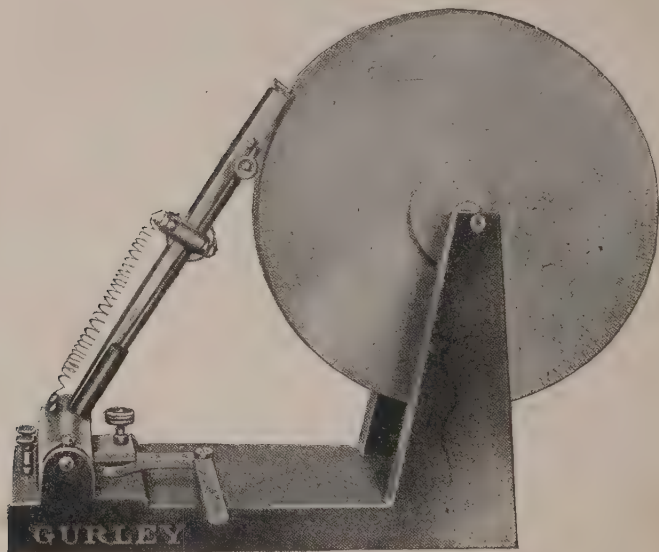


FIG. 9.

surface. A moment before the descending weight reaches the floor either stop the rotation of the wheel, or stop the vibration of the tuning fork.

When a good trace has been obtained on the wheel place a transverse mark with the aid of a scribe and try-square upon every fiftieth wave crest until 10 groups

of 50 vibrations have been counted. By means of the try-square determine the angular position of each transverse mark, reading to the nearest tenth of a degree. The rate of vibration of the fork is 128 per second, and therefore the time required to describe a group of vibrations is $t = \frac{5}{128}$ second.

With the calipers and scale measure in cm. the dimensions of the wheel and its cylindrical projection. The rotating wheel may be considered as composed of two disks placed side by side, their aggregate mass being 11,340 grams. Letting M_1 be the mass of the large disk of radius R_1 , and M_2 be the mass of the small disk of radius R_2 , then the moment of inertia of the wheel by calculation from the measured dimensions is

$$R_1 = 22.96 \div 2 = 11.48$$

$$R_2 = 5.9 \div 2 = 2.95$$

$$I = \frac{1}{2}(M_1 R_1^2 + M_2 R_2^2) \text{ gram-cm.}^2$$

OBSERVATIONS AND CONCLUSIONS. Record the angular position in degrees of each transverse mark for the 10 groups of vibrations. Derive from these readings the angular distance subtended by each group, and also the angular distance gained by each group over the next preceding one. Arrange these values in parallel columns. Compute the average angular gain in $\frac{5}{128}$ second, convert to radians, and deduce the angular acceleration α in radians per sec. per sec. Calculate the torque acting on the wheel from $\tau = 200 R_2 (g - \alpha R_2)$ dyne-cm., in which the factor $200 \alpha R_2$ is the part of the impressed force which is necessary to accelerate the descending 200-gram weight. Finally compute the moment of inertia of the wheel in gram-cm.²

Record the radius and axial length of the two disks which form the wheel, and calculate their volumes separately. Apportion the total mass of the wheel between the two disks, ignoring the hole for the shaft, and then compute the moment of inertia of the wheel from its mass and dimensions. Compare the results obtained by the two methods.

EXPERIMENT 7

Study of Harmonic Motion of Rotating System

OBJECT. To determine the moment of inertia of a body executing harmonic rotatory motion and to ascertain the influence on the vibration period of increasing its moment of inertia by the addition of weights.

THEORY. Consider a body, capable of rotatory vibration about some axis without friction, to be displaced from its position of equilibrium by an angle Φ , and then released. The resulting motion of the body will be simple harmonic and its displacement from the equilibrium position t seconds after being released will be

$$\phi = \Phi \cos \frac{2\pi t}{T},$$

where T is the time of one complete vibration. Its angular acceleration at that instant will be

$$\alpha = \frac{d^2\phi}{dt^2} = -\frac{4\pi^2}{T^2} \Phi \cos \frac{2\pi t}{T} = -\frac{4\pi^2}{T^2} \phi,$$

which shows that the angular acceleration of the body varies directly as the angle of displacement and is directed toward the position of equilibrium. If the moment of inertia of the body be I , the torque acting on it to return the body to its equilibrium position will be

$$\tau = I\alpha = -\frac{4\pi^2 I}{T^2} \phi.$$

The factor $4\pi^2 I/T^2$ is a constant, say b , for a given body executing rotatory harmonic motion, and from the foregoing equation may be defined as the torque per unit angle (radian) of deflection or twist. This constant b is readily obtained experimentally by applying a mass

m subject to gravitational force at a distance r from the axis of rotation and noting the angle of twist ϕ when the body comes to rest; then

$$b = \frac{\tau}{\phi} = \frac{mgr}{\phi},$$

where g is the acceleration of gravity. Should another body of moment of inertia I' be added to the vibrating body, its period of oscillation will be altered and assume the new value

$$T_1 = 2\pi \sqrt{\frac{I + I'}{b}}.$$

APPARATUS. Rotatory harmonic motion apparatus with small weights, two cylindrical masses, balance with weights, stop-watch, calipers, and scale.

The rotatory harmonic motion apparatus, Fig. 10, consists essentially of a pivoted body carrying a pointer that moves over a circular scale, which body is attached to one end of a flat spiral spring whose other end is fixed to the supporting frame. A small horizontal wheel of radius r is fastened near the top of the pivoted body, and a cord attached to this wheel extends over a vertical pulley to a scale pan. By placing weights on this pan and observing the corresponding angular displacements of the body, the calculation of the rotational constant b is made possible. When the body is displaced from its position of equilibrium and released, it oscillates to and fro harmonically. While the motion is damped by friction, the period of oscillation (i.e., the time of one complete oscillation) is inappreciably affected thereby.

PROCEDURE. To determine the rotational constant, apply a small weight in grams to the scale pan and measure the angle of displacement in degrees. Be careful to compensate for static friction. Then apply another weight of known mass to the first one, and measure the total angle of displacement. Repeat this process by

adding successively three other weights. Measure with the calipers the radius of the wheel which is attached to the pivoted body.

To measure the period of vibration of the pivoted body,

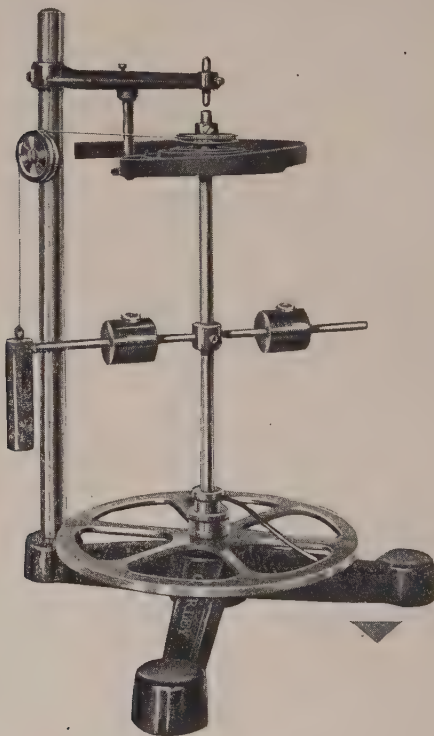


FIG. 10.

place the scale pan and cord in such position as not to interfere with its free vibration and determine with a stop-watch the time required in making ten complete vibrations. Repeat this measurement three times.

To ascertain the influence on the period of vibration of increasing the moment of inertia of the pivoted body, clamp the two cylindrical equal masses on the extremities of the horizontal rod of the body at equal distances from the axis, and determine the time required in making ten complete vibrations. Repeat this measurement three times.

Weigh the hollow cylinders and call their aggregate mass M grams. Measure in cm. the outside diameter D , the inside diameter d , and the axial length l of the cylinders. Then if R cm. be the distance from the center of either mass to the axis, the moment of inertia of the two masses about the axis of oscillation will be

$$I' = M \left(\frac{D^2 + d^2}{16} + \frac{l^2}{12} + R^2 \right) \text{ gram-cm.}^2$$

OBSERVATIONS AND CONCLUSIONS. Record all necessary data and arrange them in tabular form. Convert the angular displacements of the pivoted body with applied torque from degrees to radians. Compute the value of the rotational constant, b dyne-cm. per radian, for each displacement ϕ , and obtain its average value. From the times of 10 vibrations of the pivoted body both with and without the cylindrical masses, deduce the average periods of vibration in seconds. Calculate the moment of inertia I of the pivoted body from the period and rotational constant. Compute the total moment of inertia I' of the two cylinders from their measured dimensions and masses. Show that the observed period of vibration of the body with the cylinders attached agrees with the value calculated from the equation for T_1 . What error would be introduced in the calculated value for this period by assuming the moment of inertia of the cylinders to be $I' = MR^2$ gram-cm.²?

EXPERIMENT 8

Stretch Modulus of Elasticity

OBJECT. To verify Hooke's law and to determine the stretch or Young's modulus of elasticity of various metals.

THEORY. Hooke's law states that when an elastic body is subject to a stress below its elastic limit, that body is strained to an amount always proportional to the stress. Assume a metal rod of length L to change

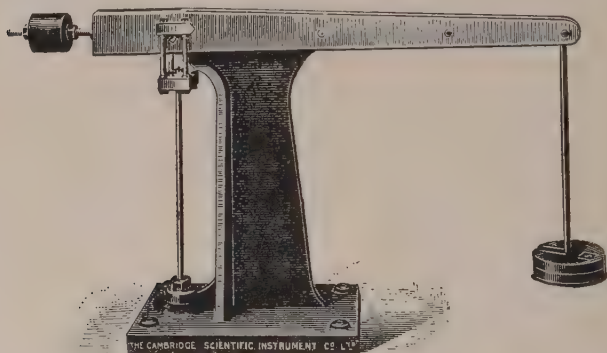


FIG. 11.

its length by l units upon the application of a force F to one end of the rod of area a . Calling the ratio of the stress to the strain the modulus of elasticity, the stretch modulus for the rod under longitudinal stress may be expressed as

$$M = \frac{\text{stress}}{\text{strain}} = \frac{FL}{al}.$$

If L and l are measured in inches, a in square inches, and F in pounds, then the modulus is expressed in pounds per square inch.

APPARATUS. Single-lever testing machine, Ewing microscope extensometer, micrometer caliper, and rods

of machine steel, Bessemer steel, wrought iron, phosphor bronze and brass.

The testing machine, Fig. 11, comprises a counter-balanced lever pivoted on a knife-edge at the top of a massive support. Near this pivot, the lever carries a block, suspended on another knife-edge, to which the upper end of the rod under test is attached, its lower

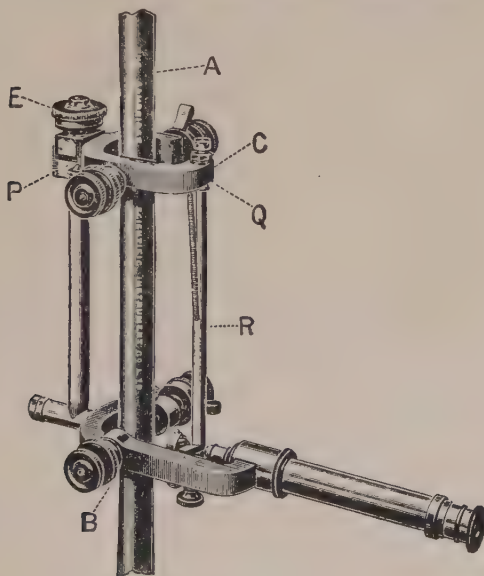


FIG. 12.

end being held in the base. The other end of the lever carries a scale pan adapted to hold weights up to 120 pounds. The lever ratio being 22.4, the stress or force per unit area on a rod of diameter D is

$$\frac{F}{a} = 22.4 \frac{4W}{\pi D^2},$$

where W is the weight on the scale pan.

Fig. 12 shows the extensometer for measuring the in-

crease of length of specimens under tensile tests. The instrument measures the extension of the rod *A* that occurs between the two pairs of opposed pointed screws at *B* and *C*, which, before the slotted bar at the rear of the extensometer was removed, were exactly 8 inches apart. When the rod is stressed, the left-hand pillar remains of constant length due to a spring at *P*, while the right-hand rod *R*, being held up against the frame *Q* by a spring, moves upward relative to the telescope. This telescope is focused on a small wire carried at the lower end of rod *R*, and the extension is observed by noting the number of small divisions *d* on the eye-piece scale through which the top of the cross-wire moves. Since each small scale division corresponds to an extension of 0.0002 inch in the 8-inch length under test, the longitudinal strain is

$$\frac{l}{L} = \frac{0.0002d}{8}.$$

The screw shown at *E* in Fig. 12 serves to bring the sighted wire to a convenient reference point on the scale.

PROCEDURE. Measure the diameter in inches of two rods at several places along their lengths with a micrometer caliper. Clamp one rod in the testing machine, carefully affix the extensometer to the rod, and then remove the slotted bar from the extensometer. Adjust the position of the mirror back of the cross-wire so that the eye-piece scale will be well illuminated. Focus the eye-piece upon the ocular scale by moving the eye-piece in or out of its tube while looking through the telescope. Then turn the small knurled screw below the telescope until a sharp focus of the upper edge of the cross-wire is obtained. To verify correctness of this adjustment, move the eye vertically while looking through the telescope and note if parallax is present, that is, if a relative movement of scale and cross-wire is discernible. If parallax is present, repeat the adjustments of the

telescope until correct. Turn screw *E* until the reading with no weights on the scale pan assumes a convenient value; then record this no-load reading. Place 20 pounds on the scale pan and take a telescope reading, estimating tenths of small divisions. Increase the load in steps of 10 pounds up to 100 pounds and observe the corresponding extensions, checking the no-load reading frequently. Remove weights, extensometer and rod and then repeat the foregoing procedure for another metal rod.

OBSERVATIONS AND CONCLUSIONS. For each sample record the material of the rod and its diameter. Place the load readings and derived values in columns bearing the following captions: Pounds load *W*, extensometer reading (in terms of small divisions), stress in pounds per square inch, strain in inches per inch, and elasticity in pounds per square inch, *M*. Calculate the average modulus of elasticity for each rod tested, by using for the stress the sum of the values in the stress column and for the strain the sum of the values in the strain column. Compare results with the values given in Table IV. Plot curve for one rod only with stress as abscissas and strain as ordinates. Explain the shape of this curve.

EXPERIMENT 9

Shear Modulus of Elasticity

OBJECT. To determine the shear modulus of elasticity (or coefficient of rigidity) of two metal rods by noting the angular displacement produced by the application of a constant torque.

THEORY. The shear (or slide) modulus of elasticity is the ratio of the shearing stress *S* to the angle of shear ϕ ; that is, the shear modulus is expressed as

$$n = \frac{S}{\phi}.$$

In the case of a rectangular parallelepiped, Fig. 13 having a horizontal face area a and a height b , and subjected to a shearing force F , the shearing stress is $S = \frac{F}{a}$, and the strain is the distortion s divided by the height b . But $\frac{s}{b}$ is the tangent of an angle ϕ , which is

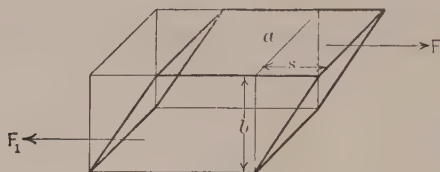


FIG. 13.

termed the angle of shear; and since this angle is usually small it is customary to write $\phi = \frac{s}{b}$. Therefore the shear modulus of elasticity for the parallelepiped becomes

$$n = \frac{Fb}{as}.$$

To apply the foregoing to a cylindrical rod subject to torsional deformation, consider a hollow cylindrical element of radius r and of thickness dr within the solid cylinder of radius R and length l , Fig. 14. With one end of this element clamped, the application of a torque $d\tau$ upon the other end will cause lines such as AB , which were originally parallel to the axis of the cylinder, to assume other positions such as AB' , making angles ϕ with their former positions. If the angular displacement of the free end be θ , then since ϕ is small, the angle of shear becomes

$$\phi = \frac{r\theta}{l}.$$

The shearing stress, or tangential force per unit area, for the element of area $2\pi r dr$ is clearly

$$S = \frac{d\mathcal{T}}{2\pi r^2 dr}.$$

Eliminating S and ϕ from the foregoing expressions, there results

$$d\mathcal{T} = \frac{2\pi\theta n}{l} r^3 dr$$

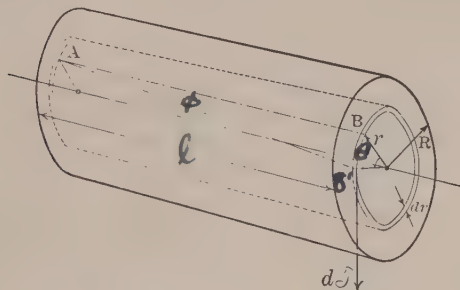


FIG. 14.

Integrating over the entire cylinder, the shear modulus for the entire solid cylinder of radius R and subject to the total torque $\mathcal{T}$ becomes

$$n = \frac{2l\mathcal{T}}{\pi\theta R^4}.$$

When l and R are expressed in cm., $\mathcal{T}$ in gram-cm., and θ in radians, then the shear modulus is expressed in grams per sq. cm.

In the apparatus used in this experiment, the angle θ is determined by attaching mirrors to the rod under test and observing the deviations of these mirrors by telescopes and scales. If the scale reading change by a cm. with the

application of torque, and d cm. be the distance between the mirror and scale, then the angular displacement of the rod and mirror is

$$\theta = \frac{1}{2} \tan^{-1} \frac{a}{d} = \frac{a}{2d} \text{ radians}$$

when the deflection is small.

APPARATUS. Shear-modulus apparatus, micrometer caliper, meter-stick, and rods of steel and brass of various diameters.

The apparatus for determining the shear modulus is shown in Fig. 15. One end of the rod to be tested is firmly clamped at the left support, and after affixing the lever arm in a horizontal position at the other end, the rod is centered by the large screw at the right support. (For small samples, bushings are used which fit the rod and supports snugly, and are inserted in the supports so that the set pins engage the corresponding holes in these bushings.) The two mirror clamps are fastened to the rod by pointed screws against V-shaped edges, the distance l between the clamps being the same as the distance between the slots of the jig used while setting the mirror clamps in place. The scales and telescopes are supported at the top of the apparatus and are placed vertically over the mirrors. To take observations on a metal rod, place the scale pan on the spring hook of the lever arm, and then adjust the inclination of the mirrors and focus the telescopes so as to yield clear images of convenient scale divisions on the cross-hairs of the telescopes. Upon placing weights on the scale pan both mirrors will rotate, one more than the other, the difference of their scale deflections being the twist a .

PROCEDURE. Measure the diameter of the rod with the micrometer caliper at four different points along its length. Measure with the meter-stick the length of the lever arm, the distance between mirror clamps, and the distance from each mirror to its scale. After adjusting

the mirrors, scales and telescopes into their proper positions, take a zero reading through the telescope. Then with 5, 10, 15, 20 and 25 pounds on the scale pan note the telescope readings. Make measurements on

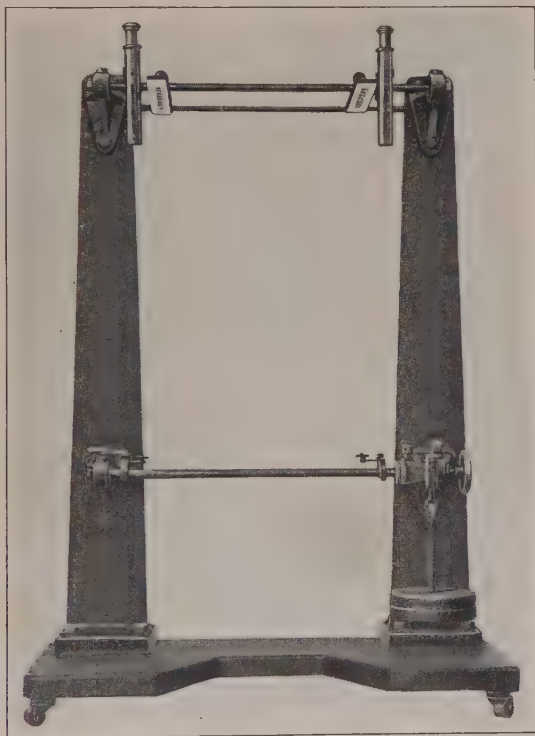


FIG 15.

two rods either of different materials and of the same diameter, or of the same material but of different diameters.

OBSERVATIONS AND CONCLUSIONS. Record observations and results for each rod as follows:

Material _____

Diam. of Rod

(in.) Length of lever arm _____ cm.

_____ Distance from mirrors to scale, d _____ cm._____ Distance between mirror clamps, l _____ cm._____ Average radius of rod, R _____ cm.

Load in pounds	Torque in gram- cm. T	Scale Readings (cm.)				Twist α	Angular dis- place- ment in radians θ	Shear Modulus in grams per sq. cm. n
		Left mirror		Right mirror				
		Read- ing	Deflec- tion	Read- ing	Deflec- tion			
0								
5								
10								
15								
20								
25								

Compute the average shear modulus for each rod from the equation

$$\left[n = \frac{2l\Sigma\mathcal{T}}{\pi R^4\Sigma\theta} \right]$$

where $\Sigma\mathcal{T}$ and $\Sigma\theta$ are the sums of the values in the columns headed $\mathcal{T}$ and θ respectively. Express the result also in pounds per square inch. Compare results with the values given in Table IV.

EXPERIMENT 10

Specific Viscosities of Liquids

OBJECT. To determine the specific viscosities of water at several temperatures by Coulomb's method.

THEORY. Viscosity of a liquid is the resistance which it offers to changes of shape and is due to molecular friction between adjacent layers in the liquid which are

moving with different velocities. If a metal disk, which is wetted but not chemically acted upon by the liquid under test, is suspended by means of a long thin wire and set into rotatory vibration while immersed in that liquid, the various layers of liquid above and below the disk will be set in motion to different extents, and the disk will have its amplitude of vibration continually reduced. The liquid, due to its viscosity, exerts a retarding force on the disk which is proportional to the velocity of the vibrating disk.

The motional equation of a body describing harmonic motion in a straight line and subject to a retarding force that is proportional to its velocity is

$$\frac{d^2s}{dt^2} + 2k \frac{ds}{dt} + \frac{a}{r} s = 0,$$

where s is the displacement of the body from its position of equilibrium at the time t reckoned from that position, $2k$ is a constant depending upon the retarding force, a is the maximum acceleration of the body, and r is its amplitude of vibration. The solution of this equation, found by placing $s = \epsilon^{mt}$, is

$$s = A\epsilon^{m_1 t} + B\epsilon^{m_2 t},$$

where A and B are constants as yet undetermined, ϵ is the base of the natural system of logarithms, and for oscillatory motion

$$m_1 = -k + j\alpha \quad \text{and} \quad m_2 = -k - j\alpha,$$

in which $\alpha = \sqrt{\frac{a}{r} - k^2}$ and $j = \sqrt{-1}$. Introducing these values of m_1 and m_2 and converting the imaginary exponential terms into their equivalent trigonometric forms by means of $\epsilon^{\pm j\theta} = \cos \theta \pm j \sin \theta$, there results

$$s = \epsilon^{-kt} [(A + B) \cos \alpha t + j(A - B) \sin \alpha t].$$

Reckoning time and displacement from the position of equilibrium, it follows that $s = 0$ when $t = 0$; whence $A + B = 0$. Therefore

$$s = (A - B)j\epsilon^{-kt} \sin \alpha t.$$

By considering the time intervals elapsing between successive occasions when the velocity of the body is zero, it is seen that $\alpha = \frac{2\pi}{T}$, where T is the time of one complete vibration. If C be the amplitude when $t = \frac{T}{4}$, then at any time t the displacement is

$$s = C\epsilon^{k(\frac{T}{4} - t)} \sin \frac{2\pi t}{T}.$$

When $t = \frac{T}{4}$, $t = \frac{5}{4}T$, $t = \frac{9}{4}T$, etc., the amplitudes of vibration will be respectively

$$\begin{aligned} s_1 &= C, \\ s_2 &= C\epsilon^{-kT} \\ s_3 &= C\epsilon^{-2kT}, \text{ etc.} \end{aligned}$$

Therefore the ratio of a linear displacement to the next preceding one in the same direction has the constant value of

$$\frac{s_3}{s_2} = \frac{s_2}{s_1} = \epsilon^{-kT}.$$

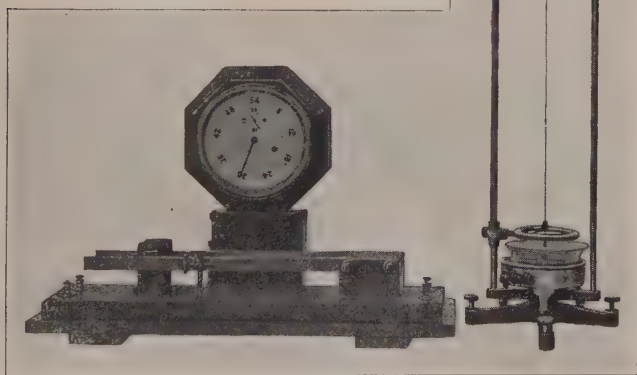
This result also applies to a body executing damped rotatory harmonic motion. Thus, if δ represent the mean ratio of an arc of oscillation of a disk immersed in a viscous liquid to its next preceding arc of oscillation in the same direction, then the ratio of damping is

$$\delta = \epsilon^{-kT},$$

where k is a constant proportional to the retarding force, which in turn depends upon the viscosity of the liquid. The value of this constant, which may be termed the relative viscosity of the liquid, is obtained from the foregoing equation as

$$k = - \frac{2.3026}{T} \log_{10} \delta.$$

Consequently the logarithm of the ratio of damping of a disk oscillating in liquids serves as a measure of their viscosity.



APPARATUS. Viscosimeter, and stop-watch or electrical clock, shown above.

The viscosimeter consists of a brass disk suspended by a thin vertical wire in a liquid contained in a dish and which can be warmed by an electric heater. The disk is provided with a pointer which plays over a stationary graduated circle, thus enabling the measurement of the vibration amplitude of the disk.

PROCEDURE. Place ice water in the viscosimeter dish and measure the temperature of the water. Displace the disk 180 degrees from its position of equilibrium

and release it, exercising care to have the disk oscillate around its own center. Observe the indications of the pointer at the ends of nine successive complete oscillations. Again measure the temperature of the water. Displace the pointer and note the time of ten complete vibrations. Repeat these operations for water at about 10° , 20° and 40° C. Between tests heat the water slowly and avoid the formation of bubbles on the disk.

The relative viscosities of other liquids at various temperatures may be determined in the same manner.

OBSERVATIONS. Record observations of experiment for each of the four tests as indicated below:

Viscosimeter Readings

No. of Oscillation	TEST 1	
	Degrees Amplitude of Disk	Ratio of Damping δ
1.....	—	—
2.....	—	—
3.....	—	—
4.....	—	—
5.....	—	—
6.....	—	—
7.....	—	—
8.....	—	—
9.....	—	—
10.....	—	—

Average ratio of damping, $\delta =$ —

Temperature of water, $^{\circ}\text{C.}$ { at start —
at end —
mean —

Time of 10 complete vibrations of disk — sec.

Time of 1 complete vibration of disk, $T =$ — sec.

CONCLUSIONS. Compute the relative viscosities of water at the four temperatures and plot the results in the form of a curve. Compare these values with the specific viscosities of water given in Table V.

EXPERIMENT 11

Conformity of Air with Boyle's Law

OBJECT. To determine how closely air at room temperature agrees with Boyle's law over the range from half to double atmospheric pressure.

THEORY. The volume v of a perfect gas at constant temperature varies inversely as the pressure p upon the gas, or symbolically, $p v = \text{constant}$. All gases deviate somewhat from this law, especially near their regions of liquefaction.

APPARATUS. Boyle's law apparatus, aneroid barometer, and thermometer.

The apparatus used to illustrate the relation between the volume and pressure of a gas consists of a U-tube formed by two straight glass tubes connected at the bottom by a flexible rubber hose. The two glass tubes are mounted vertically on the frame and may be raised and lowered along a graduated scale and clamped in any desired position. One glass tube is graduated to read volumes directly in cubic centimeters, and is also provided with a stop-cock at its upper end. Mercury is placed in the U-tube and the gas to be investigated is confined in the graduated tube above the mercury surface. The heights of the mercury on both sides of the tube can be measured by a slider carrying a vernier which moves along the central scale. The index of the slider is a horizontal scratch on a mirror which extends back of each glass tube. In taking a reading on either column, the slider is moved until the image in the mirror of the pupil of the observer's eye, the scratch on the mirror, and the top of the mercury column are seen to lie in one straight line.

The aneroid barometer consists of an air-tight cylindrical chamber, the cover of which moves slightly in or out with changes of atmospheric pressure. This motion is enhanced by a delicate system of levers and is trans-

mitted to a pointer which moves over a properly calibrated scale.

PROCEDURE. After the apparatus has been leveled, entrap 6 cu. cm. of air in the graduated tube at atmospheric pressure. The stop-cock should then be kept closed until all observations have been made. Take a reading of the height of mercury in each tube. Increase the pressure by raising the open tube or lowering the graduated tube in such increments that the volume of enclosed air will be successively 5.5, 5.0, 4.5, 4.0, 3.5 and 3.0 cu. cm., taking readings of mercury elevations in both tubes in cm. at each volume. Then decrease the pressure and observe the height of the mercury columns for enclosed gas volumes successively of 6, 7, 8, 9, 10, 11 and 12 cu. cm. Take readings of the thermometer and barometer at frequent intervals during the experiment.

Variations in temperature of the entrapped air affect the accuracy of the experiment greatly; in fact, a rise of 1° C. at room temperature introduces a change of about one-third of 1 per cent in the value of p_v . Therefore avoid touching the tube containing the air under test. After each change of volume wait a minute or two before taking the readings of pressure and volume—why?

OBSERVATIONS AND CONCLUSIONS. Record the observations of mercury elevations in the open and closed tubes for the various air volumes in tabular form. In another column record the absolute pressures (p) in cm. Hg. of the confined gas, obtained by adding to or subtracting from atmospheric pressure, expressed in cm. Hg., the differences in elevation of the two mercury columns. In the last column insert the computed products of volume and absolute pressure for the different volumes of air. Plot two curves: one of pressure p in cm. Hg. against volume v in cu. cm., and the other of p_v products against pressures p in cm. Hg. Determine the percentage maximum deviation of the product p_v from the value of this product at atmospheric pressure.

✓
75.4
71.0

EXPERIMENT 12

Specific Gravity of Gases with Effusimeter

OBJECT. To determine the specific gravity of illuminating gas by means of the effusimeter and to determine the rate of gas consumption of several types of gas burners.

THEORY. The operating principle of an effusimeter is that light gases effuse more rapidly through a small orifice than do denser gases. Suppose a gas, of density δ grams per cu. cm., enclosed in a chamber under a pressure p dynes per sq. cm. above that of the surrounding region, be allowed to escape through a small orifice of area a sq. cm. By equating the gain of kinetic energy of the gas per second to the loss of its potential energy per second, the speed of efflux in cm. per sec. is found to be

$$v = \sqrt{\frac{2p}{\delta}}.$$

Therefore two gases of densities δ_1 and δ_2 would effuse with velocities of v_1 and v_2 respectively, such that

$$\frac{\delta_1}{\delta_2} = \frac{v_2^2}{v_1^2}.$$

Since the velocities of efflux are inversely proportional to the respective times t_1 and t_2 required for the effusion of the same volume of the two gases through the same orifice, it follows that

$$\frac{\delta_1}{\delta_2} = \frac{t_1^2}{t_2^2}.$$

APPARATUS. Effusimeter, stop-watch or electrical clock, gas-meter, pressure gage, Bunsen, Meker and fish-tail burners.

The effusimeter, Fig. 16, consists of a U-shaped glass tube, with a detachable metal fitting containing the effu-

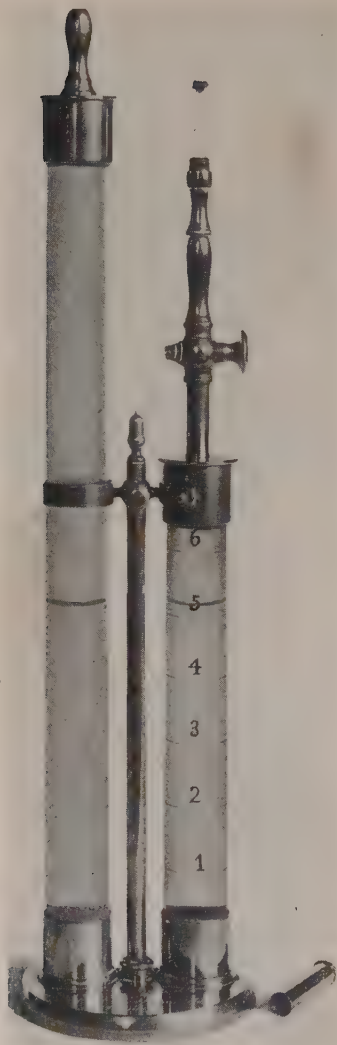


FIG. 16

sion orifice with cover at the upper end of the graduated tube, and with a suction fitting at the other or suction tube. Water is placed in the lower part of the tubes. To take an observation with the instrument, the orifice fitting is removed, connection is made with the gas supply, the stop-cock at the top of the graduated tube is opened, and suction is applied until the water in the graduated tube falls below the first division. When the gas is thus drawn into the tube, close the stop-cock and replace the orifice fitting with its cover removed. Then open the stop-cock and observe the time necessary for the water to rise through several divisions of the graduated tube.

PROCEDURE. Draw air into the graduated tube and then allow it to escape, without using the effusion orifice. Repeat this process several times, and then leave the graduated tube full of air at the last operation. With

orifice in place, note with a stop-watch the time t_1 that is required for three divisions of the air volume to be displaced by the rising water. Take six such readings on air. Repeat the foregoing operations for illuminating gas to determine the time t_2 for the same volume of gas to effuse through the same outlet. As the excess pressure p in this type of effusimeter becomes less as the water rises, the same three tube divisions must be used for all tests.

As an addenda to this experiment, determine by means of a gas meter and watch the times for 1 cubic foot of illuminating gas to be burned at a Bunsen burner, a Meker burner and a fish-tail burner, for the purpose of obtaining an idea of the relative gas consumption of these burners. Measure the pressure of the gas at the burner.

OBSERVATIONS. Make a tabular record of observations as below:

Effusimeter Readings (sec.)

Time for air to effuse	Time for illuminating gas to effuse
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
Aver. _____ = t_1	Aver. _____ = t_2

Gas Consumed by Gas Burners

Burner.	Time to burn 1 cu. ft. of illuminating gas (sec.)	Gas pressure at burner (in. of water)	Rate of gas con- sumption (cu. ft. per hr.)
Bunsen	_____	_____	_____
Meker	_____	_____	_____
Fish-tail	_____	_____	_____

CONCLUSIONS. Compute the specific gravity of illuminating gas with respect to air as unity. Taking

the density of air at 0° C. and standard pressure as 0.001293 gram per cu. cm., calculate the density of illuminating gas under room conditions.

EXPERIMENT 13

Calibration Curve of Venturi Meter

OBJECT. To determine the quantities of water discharged through a Venturi meter under different heads, and to compare these values graphically with the theoretical discharges of incompressible, frictionless liquids.

THEORY. The Venturi meter depends for its operation upon the fact that the pressure in a constricted portion or throat of a tube through which a liquid flows is less than the pressure in the unconstricted portion. The total energy (kinetic and potential) of unit volume of liquid of density δ flowing through the tube at velocity v under pressure p (or head h) is

$$W = \frac{1}{2}\delta v^2 + p = \frac{1}{2}\delta v^2 + \delta gh,$$

which is the same for a frictionless liquid at all points of the tube. If the velocity of the liquid be v and the head be h where the cross-sectional area of the tube is a , and the velocity be v_1 and the head be h_1 where the constricted area is a_1 , then

$$v^2 + 2gh = v_1^2 + 2gh_1.$$

Also, in an incompressible liquid, the rates of discharge of the liquid are the same at all cross-sections, whence

$$av = a_1v_1.$$

Eliminating v_1 from the last two equations and solving

for the discharge $Q = av$, there results as the theoretical discharge

$$Q = aa_1 \sqrt{\frac{2g(h - h_1)}{a^2 - a_1^2}} = K \sqrt{h - h_1},$$

where the discharge constant $K = aa_1 \sqrt{\frac{2g}{a^2 - a_1^2}}$. If

a and a_1 are expressed in sq. cm., h and h_1 in cm., and g in cm. per sec. per sec., then Q will be expressed in cu. cm. per sec.

APPARATUS. Venturi meter connected with water supply, vessel for collecting water, graduate, and watch.

The Venturi meter consists of a horizontal tube with a constricted portion or throat and two vertical pressure tubes, one of which connects with the tube and the other with the throat. When water is passed through the meter, some water flows into the pressure tubes, and the heights to which it rises in the two tubes indicate the pressure heads of the water in the horizontal tube and in the throat. The theoretical discharge of water through the meter is a function of the difference of height of water in the pressure tubes.

PROCEDURE. Turn on the water through the meter, and before taking observations, see that all bubbles have been removed from the unconstricted portion of the meter having the pressure tube. When fairly constant conditions have been secured, direct the outflowing water into the collecting vessel for a few minutes. Observe the heights of the water in pressure tubes during the discharge, note the time of discharge, and measure with the graduate the total amount of water collected during the test. Repeat the foregoing operations with different pressures, that is, with different values of $h - h_1$.

OBSERVATIONS. Record observations as indicated on the next page:

Constants of Venturi Meter

Cross-sectional area of tube..... (a) = ——— sq. cm.
 Cross-sectional area of throat..... (a_1) = ——— sq. cm.
 Discharge constant..... (K) = ———

Water Discharge of Meter

Water in pressure tubes (cm.)			Theoret- ical dis- charge (cu. cm. per sec.) Q	Volume of water dis- charged during test (cu. cm.)	Time of dis- charge (sec.)	Actual dis- charge (cu. cm. per sec.) Q'
Height at tube h	Height at throat h_1	Differ- ence $h - h_1$				
—	—	—	—	—	—	—
—	—	—	—	—	—	—
—	—	—	—	—	—	—
—	—	—	—	—	—	—
—	—	—	—	—	—	—
—	—	—	—	—	—	—
—	—	—	—	—	—	—

CONCLUSIONS. Calculate from the equation the theoretical discharge Q and from the tests the actual discharge Q' of water during each test, and insert these values in above table. Plot a curve with Q as abscissas and Q' as ordinates.

EXPERIMENT 14**Velocity of Sound—Specific Heats of a Gas**

OBJECT. To measure the velocity of sound in a metal rod and in air, to determine the pressure of the atmosphere and its density, and to calculate the ratio of the specific heat of air at constant pressure to that at constant volume.

THEORY. To ascertain the velocity of sound in a gas,

consider a wind of such velocity as to oppose exactly the advance of sound waves, traveling with the same velocity in the gas. Each portion of the gas assumes in turn the states of condensation and rarefaction imposed by the sound waves, but these states are stationary in space due to the opposing wind. The pressure, volume per unit mass, and the velocity of the gas particles (medium being considered at rest) vary from point to point along the direction of propagation. Let these characteristics be respectively p , w and v at one point and be p' , w' and v' at another point fixed in space. Assuming the sound waves to retain their wave-shape as they advance, there will be no accumulation nor diminution of gas between the two reference points, and therefore the masses of gas which pass these points per unit time and per unit area must be equal, or

$$\frac{v}{w} = \frac{v'}{w'}.$$

These masses have different velocities and therefore different momenta. The gain or loss of momentum of a prism of gas in unit time, due to the difference of pressure at the two reference points, is therefore

$$p - p' = \frac{v'}{w'} w' - \frac{v}{w} w.$$

Eliminating v' from the foregoing equations, and solving for v , there results as the velocity of the sound relative to the gas

$$v = \sqrt{\frac{p - p'}{w' - w}} w^2 = \sqrt{\frac{E}{\delta}}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $E = \frac{p - p'}{w' - w}$ is the bulk modulus of elasticity of the gas, and $\delta = \frac{1}{w}$ is its density.

As the compressions and rarefactions of the medium when traversed by sound waves occur with great rapidity, the heat developed by compression is not diffused, nor is heat absorbed from the surroundings during rarefactions. In consequence, the modulus of elasticity E in the foregoing expression is the volume coefficient for adiabatic conditions. This coefficient may be expressed in terms of the isothermal modulus of elasticity e and the specific heats of a gas under constant pressure and constant volume, respectively c_p and c_v , namely

$$E = e \frac{c_p}{c_v} = e\gamma, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where γ is the ratio of the two specific heats of the gas. (Refer to § 184 of Reed and Guthe "College Physics.") The modulus of elasticity under isothermal conditions can be ascertained by considering a volume V of gas at pressure p to have its volume reduced by dV under an increment of pressure dp . Then by Boyle's law

$$pV = (p + dp)(V - dV),$$

which, by neglecting the small product $dp \cdot dV$, reduces to

$$p = \frac{dp}{\frac{dV}{V}} = e. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Combining equations (1), (2) and (3), there is obtained Laplace's formula for the velocity of sound in a gas, namely,

$$v = \sqrt{\frac{p\gamma}{\delta}}. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

When p is the pressure of the gas in dynes per sq. cm. and δ is its density in grams per cu. cm., then v is the

velocity of sound in cm. per second. In order to determine γ experimentally from this equation, the factors v , p and δ must be measured, the methods of their measurement being next considered.

A metal rod of length L cm. clamped at its center is set into longitudinal vibration in its fundamental mode and its frequency of vibration is found to be f . Then the velocity of sound in the rod in cm. per second is

$$v = 2fL, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

since the length of the rod is one-half of a wave-length. If one end of the rod project into a closed tube of proper length and that end be provided with a light disk which fits loosely into the tube, then the rod, when placed in vibration, will set up stationary waves in the gas enclosed in the tube. Cork filings strewn inside the tube will collect at points where the gas particles have their greatest amplitudes. The average distance l cm. between centers of successive groups of the filings will be half of a wave-length of the sound in the gas; whence the velocity of sound therein is

$$v = 2fl = v_r \frac{l}{L} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

cm. per second at the temperature at which the experiment is performed.

The pressure of a gas at any temperature may be determined by measuring the height h cm. of a column of mercury that it can support. Thus the pressure in dynes per sq. cm. is

$$p = h d g, \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where d is the density of the mercury in grams per cu. cm. and g is the acceleration of gravity in cm. per sec.² If air at atmospheric pressure be under investigation

the value of h can be read from a mercurial or an aneroid barometer.

When several gases are mixed, the total pressure of the mixture is equal to the sum of the pressures exerted by the component gases individually. Thus, atmospheric pressure is made up of the pressure of dry air and that of the water vapor present. If these component pressures be respectively p_a and p_w , then atmospheric pressure is

$$p' = p_a + p_w \text{ cm. of Hg.}$$

The value of p_w may be determined from measurements of the dew-point (see Experiment 21) in connection with Table XII. The densities δ_0 of most gases are known at 76 cm. of Hg. (p_0) and 0° C. from carefully performed experiments; the values for a few gases are given in Table VI. The density of a dry gas at any other pressure p_a and temperature t is

$$\delta_a = \delta_0 \frac{p_a}{p_0} \cdot \frac{273}{273+t}.$$

Taking the density of dry air at 76 cm. of Hg. and 0° C. as 0.001293, and the density of water vapor relative to air at the same temperature as $\frac{\delta_w}{\delta_a} = 0.62$, the density of the atmosphere at temperature t° C and pressure p cm. of Hg. is

$$\begin{aligned} \delta &= \delta_a + \delta_w = 0.001293 \left(\frac{p_a}{76} + \frac{0.62p_w}{76} \right) \frac{273}{273+t} \\ &= 0.004645 \frac{p' - 0.38p_w}{273+t} \quad \dots \quad (8) \end{aligned}$$

where the pressure of water vapor p_w is also expressed in cm. of Hg.

Having determined the value of v from equation (6), p from equation (7) and δ from equation (8), these values

are substituted in equation (4), which can be rewritten in the form

$$\gamma = \frac{v^2 \delta}{p},$$

in order to find the ratio γ of the specific heat at constant pressure to that at constant volume.

APPARATUS.—Kundt's tube with metal rod, toothed wheel driven by a motor rotator, motor rheostat, tachometer or speed indicator, aneroid barometer, hygrometer and thermometer.

The Kundt's tube is a glass cylindrical tube provided with metal fittings at its ends and mounted in a hori-

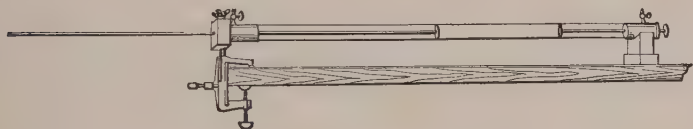


FIG. 17.

zontal position as shown in Fig. 17. A metal rod is inserted into the tube and clamped at its middle point at the left-hand fitting. At the other end of the tube a tight-fitting stopper can be moved in or out and be clamped in any desired position. In this way the length of the gas column between the end of the rod and the stopper can be properly adjusted. Measurements on many gases at various temperatures may be made with this device.

PROCEDURE AND OBSERVATIONS.—Clamp the metal rod at its center and set it into longitudinal vibration by rubbing one end of the rod in the direction of its length with resined chamois, endeavoring to obtain a clear and loud tone. Start the motor rotator carrying the toothed wheels and allow a spring to press against the revolving teeth of one of the wheels. Adjust the speed of the motor by manipulating the rheostat until the sound

emitted by the vibrating spring is exactly of the same pitch as that produced by the rod, and then measure this speed by means of a tachometer or a speed indicator. Knowing the number of teeth on the wheel employed in this measurement and the speed of the wheel in revolutions per second, the frequency of vibration f of the rod is obtained as their product. Then measure the length of the rod in cm. and calculate the velocity of sound in the material of which the rod is composed. Compare the result with the values given in Table XVI.

Place a disk upon one end of the rod and insert that end into the Kundt's tube and fasten the rod at its center to the clamping bushing. Distribute cork filings evenly over the bottom of the tube and then set the rod in vibration. If the filings do not show a disturbance, move the stopper in small steps in either direction until a position is found which will cause the filings to assume well-defined ridges along the tube corresponding to the position of antinodes of the waves in the air. Measure the average distance between centers of successive ridges and also observe the temperature of the room.

Observe the atmospheric pressure by reading the aneroid barometer in inches of Hg. and reduce this value to dynes per sq. cm. Determine the dew-point by means of a Daniell hygrometer or a hygrodeik, as indicated in Experiment 21, and ascertain the pressure of water vapor in the atmosphere.

CONCLUSIONS.—From the tabulated observations determine the velocity of sound in air at atmospheric pressure and room temperature. Compute from the barometric and hygrometric observations the density of the atmosphere in grams per cu. cm. Finally calculate the ratio of the specific heats of air from Laplace's formula and compare the result with the value given in Table VII.

EXPERIMENT 15

Coefficient of Expansion of Gases by Air Thermometer

OBJECT. To determine the coefficient of expansion of air by means of a constant volume air thermometer.

THEORY. Consider a gas at 0° C. to occupy a volume v_0 under a pressure p_0 , while at t° C. it occupies a volume v_t under a pressure p_t . Then according to the Boyle Gay-Lussac law for perfect gases

$$p_t v_t = p_0 v_0 (1 + \alpha t),$$

where α is the coefficient of expansion of the gas. If the volume remains unchanged while the temperature is varied by a known amount, measurements of the pressures enable the determination of the expansion coefficient, since

$$\alpha = \frac{p_t - p_0}{p_0 t}.$$

Fig. 18 shows the general design of the air thermometer employed. The gas under investigation is enclosed in the bulb A , which is connected with a U-tube containing mercury. The volume of the enclosed gas may be kept constant during a test by raising or lowering tube B so that the surface of the mercury in tube C just touches the index point I . Let the height of this index above some convenient datum be h and the heights of the mercury in tube B when the bulb is surrounded by melting ice and then by steam be respectively h_0 and h_t , as indicated in the figure. Since the pressure on the gas is proportional to the difference

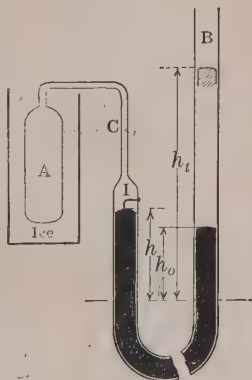


FIG. 18.

of the heights of the two mercury columns, it follows that

$$\alpha = \frac{h_t - h_0}{t(H + h_0 - h)},$$

where t is the temperature of steam, and H is the barometric height. The temperature of steam in degrees C. may be computed from the barometric height H cm. Hg. by using the formula

$$t = 100 + 0.375(H - 76).$$

The foregoing simple expressions for α are not directly applicable to the present experiment because the volume of the bulb increases when heated from 0° to t° C., and also because a small portion of the gas under test, namely that in tube C , does not change its temperature greatly from room temperature. Therefore correct expressions for the volumes v_0 and v of the gas at the respective temperatures must be obtained. Also, the pressures should be corrected for the expansion of the mercury in the columns if the temperature of these columns be higher during the observations with steam than during those with ice surrounding the bulb.

Let t_1 and t_2 be the respective temperatures in degrees C. of the mercury columns at the time when the ice and steam readings are taken, as indicated by a thermometer placed along the columns. Then the pressures reduced to 0° C. at the temperatures of 0° and t° C. are

$$p_0 = H + \frac{h_0 - h}{1 + 0.00018 t_1} = H + (h_0 - h)(1 - 0.00018 t_1),$$

approximately, and similarly

$$p_t = H + (h_t - h)(1 - 0.00018 t_2),$$

where the constant 0.00018 is the coefficient of expansion of mercury.

Let the volumes of the bulb A and the tube C at 0°C. be V and v respectively. Then the volume that the gas would occupy at pressure p_0 were its temperature 0°C. throughout is

$$v_0 = V + \frac{v}{1 + \alpha t_1},$$

and the volume which the gas would occupy at pressure p_t were its temperature $t^\circ \text{C.}$ throughout is evidently

$$v_t = V(1 + 0.000025 t) + v \frac{1 + \alpha t}{1 + \alpha t_2},$$

where 0.000025 is the coefficient of expansion of glass. Substitute the foregoing values of v_0 and v_t in the first equation and divide through by V , there results

$$\begin{aligned} p_t \left[1 + 0.000025 t + \frac{v(1 + \alpha t)}{V(1 + \alpha t_2)} \right] \\ = p_0 \left(1 + \frac{v}{V(1 + \alpha t_1)} \right) (1 + \alpha t). \end{aligned}$$

Rather than seek a general solution for α , this equation may be put in the more convenient form

$$\alpha = \frac{p_t \left(1 + 0.000025 t + \frac{v}{V(1 + \alpha t_2)} \right) - p_0 \left(1 + \frac{v}{V(1 + \alpha t_1)} \right)}{t \left[p_0 \left(1 + \frac{v}{V(1 + \alpha t_1)} \right) - p_t \frac{v}{V(1 + \alpha t_2)} \right]},$$

in which α appears also in the right-hand member, but only in the small correction factors involving $\frac{v}{V}$. Having ascertained the values of p_0 , p_t , t , t_1 and t_2 by experiment, choose some reasonable value for α and insert it in the factors $(1 + \alpha t_1)$ and $(1 + \alpha t_2)$ and then solve for α by the last equation. Thereafter introduce this com-



FIG. 19.

puted value in the correction factors and again solve the foregoing equation, this time obtaining the final value for the coefficient of expansion of the gas.

APPARATUS. Air thermometer, aneroid barometer, mercury thermometer, ice and steam baths, Bunsen burner, and tripod.

The air thermometer to be used is shown in Fig. 19. It consists of a thin cylindrical glass bulb which is joined to the chamber having the index wire by means of a glass tube of small bore. This chamber communicates through a stop-cock and a flexible hose with a straight open glass tube, both glass parts being attached to metal fittings which can be clamped to the vertical guide rods at any desired heights. The height of the bulb should remain the same throughout this experiment. An amount of mercury is placed in the tube which is sufficient to bring the mercury surfaces in the two columns to suitable heights. The heights of the mercury surfaces in any test are obtained by means of a slider which moves along a vertical scale graduated into mm., the slider carrying a vernier and a mirror with a horizontal scratch. When the slider is properly set to indicate the height of a mercury column, the image in the mirror of the observer's eye, the scratch on the mirror, and the top of the mercury column should be seen in one line. In this air thermometer the ratio $v/V = 0.024$.

PROCEDURE. Observe the height h of the wire index. After the bulb has been filled with dry air, place the vessel of the ice bath around the bulb of the air thermometer and carefully drop small pieces of cracked ice into the vessel until the bulb is completely surrounded by ice. After the air ceases to contract, a process requiring about ten minutes, adjust the level of the open column so that the mercury in the other column will just touch the wire index; then observe the height of the mercury h_0 in the open column. Repeat this observation by making another independent setting of the slider. Record the reading t_1 of a mercury thermometer supported between

the two columns and with its bulb near the wire index. Take also a reading of the aneroid barometer.

Remove the ice bath carefully and replace it by the steam bath. Light the gas under it and then put the cover in place. As steam forms around the bulb raise the open tube so as to maintain the mercury surface in the other tube near the reference point. When the air ceases to expand, that is, when the gas in the bulb has reached the temperature of steam, $t^\circ \text{C.}$, adjust the mercury exactly on the wire index and note the height h_1 of the mercury in the open column. Readjust and take another observation. Record the temperature t_2 now indicated by the mercury thermometer. Then remove the vessel and lower the open column so that the mercury cannot enter the bulb when the gas contracts in assuming room temperature.

OBSERVATIONS AND CONCLUSIONS. Reduce the barometer reading to cm. Hg. and compute the gas pressures at 0° and $t^\circ \text{C.}$ Calculate the temperature of steam t under existing conditions and finally determine the coefficient of expansion of air. Compare this result with the theoretical value given in Table VI.

EXPERIMENT 16

Specific Heats of Solids

OBJECT. To determine the specific heat of copper by the method of mixtures.

THEORY. A mass of m grams of copper is heated to a temperature $t_1^\circ \text{C.}$ and then dropped into M grams of water at $t_2^\circ \text{C.}$ contained in a calorimeter whose water equivalent is k . If the resulting temperature be $t^\circ \text{C.}$, then the specific heat of copper will be

$$s = \frac{(M + k)(t - t_2)}{m(t_1 - t)},$$

(Sp. Heat Cu = 0.093)

if radiation be ignored or be compensated for as indicated later on.

In the experiment it will be found impracticable to have the thermometer, which is used in measuring the temperature of the heated copper, entirely exposed to the same temperature as the bulb. Consequently the thermometer will read too low, and a correction for stem exposure should be made. If the reading of the thermometer be $t' ^\circ \text{C.}$, the temperature of the exposed stem as determined by another thermometer be $t_s ^\circ \text{C.}$, and the length of the exposed mercury thread in degrees be n , then the corrected reading of the thermometer (or the true temperature of the copper) will be

$$t_1 = t' + 0.000156n(t' - t_s) + c,$$

where the numerical constant is the coefficient of apparent expansion of mercury in glass per degree C., and the constant c is the calibration correction arising from irregularities in the diameter of the thermometer bore.

APPARATUS. Calorimeter and steam jacket mounted on convenient stand, boiler, balance with weights, Bunsen burner, bundle of copper scraps, three mercury thermometers, and tripod.

The calorimeter, steam jacket and protecting shutter are shown in Fig. 20. The device illustrated includes a polished calorimeter which is supported by strings within another polished metal vessel that in turn is placed in a wooden box. This box carries a support for holding the thermometer used to record the temperature of the calo-

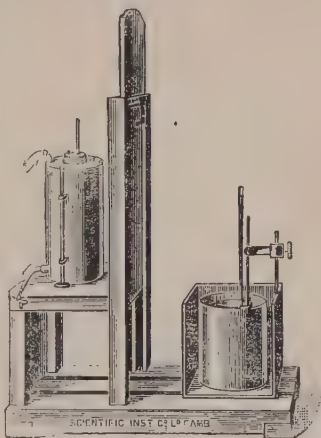


FIG. 20.

rimeter contents. The heater consists of a steam jacket which surrounds a cylindrical chamber that is open at its lower end but is closed at the top by a removable cover. The copper scraps to be measured for specific heat are suspended in the heating chamber by means of a string attached to the cover. The temperature of the copper is determined by a thermometer which is inserted through the cover to the proper depth so that its bulb will be alongside the copper scraps. To transfer the specimen from the heater to the calorimeter, the shutter is raised, the calorimeter is moved directly under the heating chamber, the string holding the copper is released and the copper is then quickly lowered into the calorimeter.

The mass of the calorimeter used is 126.5 grams and its specific heat is 0.094. The approximate volume of the immersed part of the calorimeter thermometer is 1 cu. cm., and the product of specific heat and density both for mercury and for glass is nearly 0.46. Therefore the water equivalent of the calorimeter and thermometer is

$$k = 126.5 \times 0.094 + 0.46 = 12.35 \text{ grams.}$$

PROCEDURE. Weigh the bundle of copper scraps and then suspend it in the heating chamber. After partially filling the boiler with water, ignite the gas at the Bunsen burner beneath the boiler. Pass the issuing steam through the steam jacket and from there to a condensing vessel, meanwhile keeping the calorimeter protected from the heater by lowering the wooden shutter. When the temperature of the heater chamber has been constant for some minutes, pour about 250 grams of water into the calorimeter, the water having a temperature approximately 3° C. below that of the room. Observe the temperature indicated by the thermometer located in the heater chamber, then measure the temperature of its exposed stem, and thereafter note the reading of the thermometer in the calorimeter, estimating tenths of

divisions in each case. Immediately raise the shutter, slide the calorimeter under the heater, lower the bundle of copper scraps into the calorimeter quickly but without splashing any water, withdraw the calorimeter, lower the shutter, slowly agitate the water by raising and lowering the copper by its suspending thread, and note accurately the final temperature of the water. It will be observed that the final temperature is roughly as much above room temperature as the latter was above the temperature of the calorimeter and contents just prior to the introduction of the copper; thus the gain of heat by absorption will in a large measure neutralize the loss of heat by radiation.

OBSERVATIONS AND CONCLUSIONS. Record observations in tabular form, correct the thermometer readings, and calculate the specific heat of copper. Compare this result with the value given in Table VIII.

EXPERIMENT 17

Heat Equivalent of Electrical Energy

OBJECT. To ascertain the mechanical equivalent of heat by the electrical method, or, in other words, to determine the heat equivalent of electrical energy.

THEORY. An electric heater is placed in a calorimeter containing water and the electrical energy expended as well as the heat developed is measured. The ratio of these quantities is the mechanical equivalent of heat. If the heater takes I amperes at an electromotive force of E volts for T seconds, the electrical energy expended is EIT joules. Let the mass of water in the calorimeter be M grams, the water equivalent of the heater and calorimeter be k grams, and the temperature rise be $t^{\circ}\text{C.}$, then the mechanical equivalent will be

$$J = \frac{EIT}{t(M + k)} \text{ joules per calorie.}$$

The temperature rise t is obtained from the temperature t_1 of the calorimeter and contents at the instant when the current is turned on, the temperature t_2 thereof at the moment when the current is turned off, and the correction factor which takes care of radiation. If r_1 and r_2 be the rates of rise of temperature in degrees per second of the calorimeter respectively immediately before the current is turned on and just after it is turned off (rates are considered negative if temperature falls), t_r be the average room temperature, and t_a be the average temperature of the calorimeter while the current flows through the heater, then the corrected temperature rise may be expressed as

$$t = t_2 - t_1 + \frac{(r_1 - r_2)T(t_a - t_r)}{t_2 - t_1}.$$

It will be observed that when $t_r = t_a$ the temperature rise is simply $t_2 - t_1$, for then the radiation correction reduces to zero.

APPARATUS. Calorimeter, electric heater, series resistance, voltmeter, ammeter, two thermometers, balance with weights, and watch.

The heater consists of a coil of wire within a metal tube and is designed to carry a current of about 2 amperes. It must be connected in series with a resistance when supplied with current from 110-volt supply circuits. The heater must be wholly immersed in water before current is allowed to flow through it, otherwise there is likelihood of burning out its coil. The specific heat of the heater is 0.10, and of the calorimeter and stirrer is 0.09.

PROCEDURE. Weigh the heater and the empty calorimeter separately. Nearly fill the calorimeter with a measured amount of water having a temperature of from 5 to 10° C. Suspend the heater in the water in a vertical position, but have its top project about an inch above the water surface so that no water can reach the connection terminals. Take readings of calorimeter temperatures at exactly one-minute intervals for 35 minutes,

stirring the water continuously with the stirrer. One-half minute after the eighth reading turn on the current through the heater and exactly 20 minutes later turn off the current. Voltmeter and ammeter readings should be taken at one-minute intervals midway between thermometer readings. Several observations of room temperature are to be made during the test. All thermometer readings must be made to tenths of a degree C.

OBSERVATIONS. Before making the test, prepare a table of five columns having the following captions: time in hours and minutes, temperature of calorimeter, temperature of room, voltmeter reading, and ammeter reading. After taking and recording the observations, determine the average room temperature, and the average readings of the voltmeter and ammeter. Compute the mean temperature of the calorimeter, t_a , during the time that current flows through the heater, by adding all temperatures of the calorimeter taken during that time and dividing by the number of readings.

CONCLUSIONS. Calculate the water equivalent of the calorimeter and heater. Plot the observations of calorimeter temperatures against time as abscissas, draw straight lines through the points representing the conditions before current flows through the heater and those after it ceases to flow, and connect the other points by a suitable curve. Determine from this graph the temperatures t_1 and t_2 at the junctions of the three lines, and also the rates of heat exchange r_1 and r_2 . Show room temperatures on the same curve sheet. Apply instrument corrections to the readings of the thermometers and of the electrical instruments. Compute the temperature rise of the calorimeter corrected for radiation and absorption, and then determine the heat equivalent of electrical energy.

EXPERIMENT 18

Mechanical Equivalent of Heat

OBJECT. To determine the mechanical equivalent of heat in converting mechanical energy into heat by friction.

THEORY. In the apparatus employed in this experiment heat is developed by the friction between a rotating cylindrical drum of thin brass and a silk belt wound around the drum and held approximately stationary in position by means of adjustable weights fastened to the ends of the belt. The drum, or calorimeter, contains a measured amount of water and the heat developed by friction is determined by observing the temperature rise of the calorimeter and contents. Let the diameter of the drum at its rubbing surface be d cm., the net weight holding the belt in equilibrium be M grams, and the number of revolutions of the drum to produce a given temperature rise be n ; then the mechanical work done in ergs is

$$W = \pi dnMg,$$

where the acceleration of gravity g is expressed in cm. per sec. per sec. If the calorimeter contains m grams of water and the water equivalent of the drum is k grams, and if the temperature rise be t° C., then the heat developed in calories is

$$H = (m + k)t;$$

whence the mechanical equivalent of heat is

$$J = \frac{\pi dnMg}{(m + k)t10^7} \text{ joules per calorie.}$$

APPARATUS. Callendar's mechanical-equivalent apparatus with thermometer, weights and electric motor, a 250-cu. cm. standard flask, thermometer, outlet nozzle, funnel and beaker.

Callendar's apparatus, Fig. 21, consists of a calorimeter drum mounted on a horizontal axis for rotation either by hand or by an electric motor. A silk belt is wound around the drum to form one and a half turns, and a weight holder is attached to each end of the belt. A mass of 2 or 4 kilograms is placed on the rear weight holder and one or more small masses of 25 or 50 grams, are

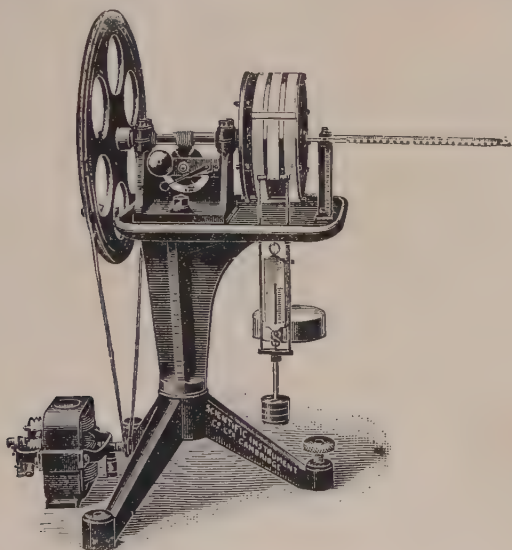


FIG. 21.

placed on the front weight holder as required to maintain floating equilibrium of belt and weights during the rotation of the drum. A spring balance fastened to the horizontal stage and to the front weight holder automatically maintains equilibrium by acting in opposition to the front weight. When the friction between the belt and drum increases the large rear weight is lifted and the front weight holder and weights descend, thereby placing a part of their weight upon the spring balance

and thus lessening the belt tension and decreasing friction. The weights of the front and rear weight holders are the same. Let p and P be the respective masses in grams of the weights placed on the front weight holder and of the large weights placed on the rear weight holder. Then, if q be the average spring balance reading, the net weight acting on the belt is

$$M = P - (p + q \text{ grams}).$$

A revolution counter is provided to register the number of turns of the calorimeter. The rise of temperature of the calorimeter and contained water is observed by means of a mercury thermometer, inserted through the central opening of the drum, whose bulb is bent around so that it will be fully immersed in the water. The water equivalent of the calorimeter and thermometer is 35.6 grams, and the diameter of the drum is 15.0 cm.

PROCEDURE. Insert the outlet nozzle in the end of the drum near the rim and verify the fact that the drum contains no water. Put the silk belt in position, place 2 or 4 kilograms on the rear weight holder, and place several small weights on the front holder, the number being adjusted by trial so that on turning the drum at a moderate speed the weights will be held in floating equilibrium between their stops. Measure out 250 cu. cm. of cold water in the standard flask, the temperature being about 6° C. below room temperature, and pour this water into the calorimeter drum. Carefully insert the bulb of the curved thermometer in the axial opening of the drum and fasten the thermometer in its support so that the bulb will be immersed in the water. Start the motor with 110 volts impressed upon its field circuit but with a lower voltage on the armature, this voltage being regulated by a rheostat in series with the armature. As soon as steady running is established, the experiment is begun by making simultaneous readings of the revolution counter and the calorimeter thermometer. Thereafter at fre-

quent intervals read the spring balance and occasionally observe the temperature of the room. Should it be necessary during the course of the experiment to vary the weights on the front holder to maintain equilibrium, the temperature intervals during which the several weights act should be noted so that a weighted average for p may be computed. When the curved thermometer indicates a temperature of the water as much above the average room temperature as it was below this value when the first reading was taken, read the revolution counter. Then stop the motor, detach the thermometer, and remove the water from the calorimeter.

OBSERVATIONS AND CONCLUSIONS. Record the values of the rear weight P and the front weight p . Determine the average reading of the spring balance, q , the number of revolutions of the drum, n , and the temperature rise of the calorimeter and contents, t . Compute the amount of work done, the amount of heat developed, and lastly the mechanical equivalent of heat.

EXPERIMENT 19

Heat of Fusion of Ice

OBJECT. To determine the heat of fusion of ice with the Bunsen ice calorimeter.

THEORY. The Bunsen ice calorimeter consists of a peculiarly shaped glass vessel and tube as shown in Fig. 22. The glass vessel consists of an inner receiving tube I open at the top and surrounded by a chamber C filled with distilled water. The lower part of this chamber is occupied by mercury, which extends up the vertical tube U and into the long horizontal calibrated tube T . The entire chamber and vertical tube are kept at 0° C. by being placed in crushed ice contained in an earthenware vessel V . By placing ether in the inner tube I and then evaporating it by forcing air from a blower through it, some of the water in C will freeze around the inner tube.

Since the density of ice is less than that of water, the formation of ice will cause some mercury to flow out the end of tube *T*. When the end of the mercury column in this tube remains stationary the ice calorimeter is ready for use.

If an electric heating coil be placed in the inner tube for *t* seconds some of the ice in *C* will melt and the end of the mercury thread will retreat say *d* divisions. Let one division of the tube represent a volume of *k* cu. cm., and let the current taken by the heating coil be *I* amperes

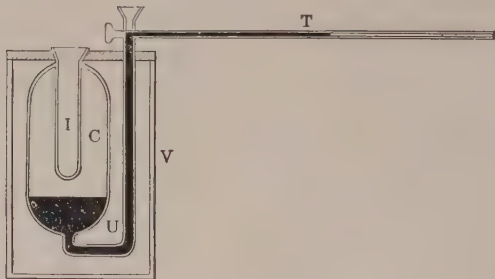


FIG. 22.

under an electromotive force of *E* volts. Then the heat developed will be

$$H = \frac{EIt}{4.2} \text{ calories,}$$

and the amount of ice melted will be

$$m = \frac{dk}{0.0907} \text{ grams,}$$

where 0.0907 is the difference of the volumes of unit masses of ice and water at 0° C. Therefore the heat of fusion of ice in calories per gram is

$$F = 0.0216 \frac{EIt}{dk}.$$

APPARATUS. Bunsen ice calorimeter, tube support, blower, ether, electric heating coil, rheostat, ammeter, voltmeter, stop-watch, funnel and ice.

The main features of the calorimeter have already been described. The junction of the vertical tube *U* and the horizontal tube *T* is effected

by an intermediate glass tube *G*, as illustrated in Fig. 23. To set up the calorimeter parts, nearly fill the enlarged upper end of tube *U* with mercury, place tube *G* firmly in position, insert graduated tube *T* so that the through-hole *a* in the attached stop-cock is vertical, pour mercury in cup *M*, slightly loosen the stop-cock in order that tube *G* will fill with mercury, turn stop-cock so that a little mercury from cup *M* will flow directly into tube *T*, and

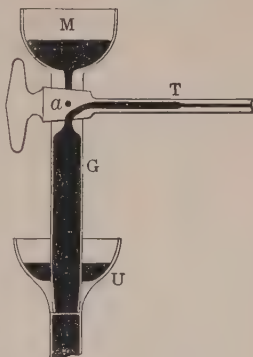


FIG. 23.

finally turn stop-cock 180 degrees to the position shown in the figure, thus completing the mercury column from the calorimeter chamber to the graduated tube.

PROCEDURE. Nearly fill the earthenware jar with cracked ice and then fill the interstices with ice-cold water. After about 15 minutes complete the mercury column to the graduated tube, and then evaporate ether in the inner receiving tube. The formation of the ice coating around this tube will be indicated by a rapid movement of the end of the mercury thread in the graduated tube. Continue the evaporation of ether until about 0.2 cu. cm. of mercury has been forced out of tube *T*, or until the ether then remaining in the receiving tube has disappeared. After the mercury in the graduated tube becomes stationary, insert the platinum heating coil in the receiving tube, and close its electrical circuit, which includes a voltmeter and an ammeter. As the end of the retreating mercury column passes some convenient

reference point, start the stop-watch. Note the indications of the voltmeter and ammeter. When the mercury thread has shortened 200 or 300 millimeter divisions, stop the watch and note the elapsed time. Quickly take another set of observations as before, and then open the circuit of the heating coil. The volume per millimeter division of tube, k , will be given.

OBSERVATIONS AND CONCLUSIONS. Record the data for the two determinations in parallel columns and compute the average value of the heat of fusion of ice. Refer to Table X.

EXPERIMENT 20

Heats of Combustion of Fuels

OBJECT. To determine the heat developed in burning illuminating gas with the Junker's continuous-flow calorimeter.

THEORY. In this determination a measured volume of gas is burned at a Bunsen burner in an air chamber which is surrounded by a water jacket, and the heat is absorbed by the water which flows in a steady stream through this jacket. Let the mass of water which passes through the jacket while v liters of gas are burned be M grams, and let the temperatures of the water on entering and leaving be t' and t° C. respectively. Water vapor is formed by the combustion of gas and subsequently condenses and leaves the chamber as m grams of water at a temperature t_1° C. Then the heat of combustion of the gas in calories per liter is

$$h = \frac{M(t - t') - m(536 + [100 - t_1])}{v}$$

where the value 536 is the heat of vaporization of water in calories per gram. In the foregoing, the volume of gas v should be the volume as measured at 0° C. and 76 cm. Hg. pressure. In the experiment the gas to be burned

is first passed through a gas meter and a pressure regulator, the latter being equipped with an open water manometer. The pressure of the gas is atmospheric pressure H cm. Hg. augmented by the height of mercury which corresponds to the difference d cm. of the water levels in the two arms of the manometer. If the gas-meter readings indicate

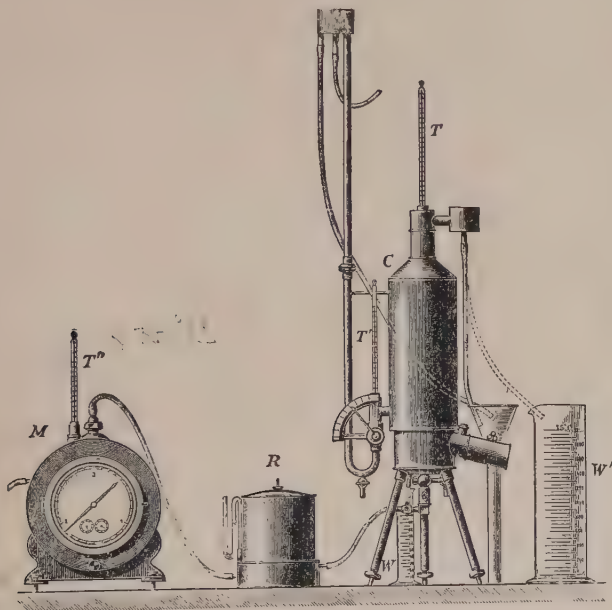


FIG. 24.

that a volume of V liters of gas has been burned, and its temperature before combustion is t''° C., then the equivalent volume at 0° C. and 76 cm. Hg. pressure will be

$$v = \frac{\left(H + \frac{d}{13.6}\right)V}{76(1 + 0.00367t'')} \text{ liters,}$$

where the constant 13.6 is the specific gravity of mercury.

APPARATUS. Junker's calorimeter, gas meter, pressure regulator, two graduates, large vessel, aneroid barometer, five thermometers, special Bunsen burner, and suitable rubber tubing.

The apparatus employed is shown in Fig. 24, in which *R* is the pressure regulator, *M* the gas meter, and *C* the calorimeter. A cross-sectional view of the calorimeter appears in Fig. 25. It consists of a combustion chamber *A*, in which the gas is burned at burner *Q*, surrounded by a water jacket *B*, which in turn is enclosed by an air jacket *L* to reduce radiation. The products of combustion pass down through a number of tubes within the water jacket and escape through the vent *Y*, while the condensed water vapor formed by the combustion of the gas passes out through the outlet *J*. Water enters a small reservoir through pipe *D* and flows down tube *E*, through regulating valve *V* and tube *F* into the water jacket, thence out through tubes *G* and *H* into the measuring vessel *W'* (Fig. 24). The supply of water is adjusted so that some water always flows over the reservoir and out the overflow tube *O*, thereby maintaining a constant head of water. The temperatures of the water on entering and leaving the calorimeter are indicated by thermometers *T'* and *T* respectively, the temperature of the gas as it reaches the burner is given by thermometer *T''*, and the temperature of the gaseous products of combustion is indicated by thermometer *T'''*. The damper *Z* is opened to provide sufficient draught for the flame.

The pressure of the gas can be increased by placing weights in the form of disks upon the dome of the pressure regulator, each 22-gram disk augmenting the pressure by 2 mm. Hg.

PROCEDURE. After examining the apparatus to make sure that it is properly assembled, turn on the water at the faucet and increase its flow until it is sufficient to produce a continuous but moderate overflow. Remove the Bunsen burner, turn on the gas at the supply main, light the gas at the burner, and reclamp it properly in

the combustion chamber. Caution—*Never have the gas burning in the calorimeter unless water is flowing through the water jacket.* Adjust the flow of water through the calorimeter by means of valve *V* and the flow of gas to the burner by means of valve *P* so that the thermometers *T''* and *T'''* indicate approximately the same temperature. After all thermometers indicate steady readings and when water drops from outlet *J*, and at the instant when the large hand of the gas meter passes the zero mark, place collecting vessels *W'* and *W* in position to collect respectively the water from outlets *H* and *J*. Take simultaneous readings of thermometers *T* and *T'* at one-minute intervals and when 9 liters of gas have been burned remove the collecting vessels. Measure the temperature of the condensed water vapor collected in vessel *W*.

Measure the mass of the water collected in each vessel. Perform the experiment a second time, and thereafter turn off the gas and *then* the water.

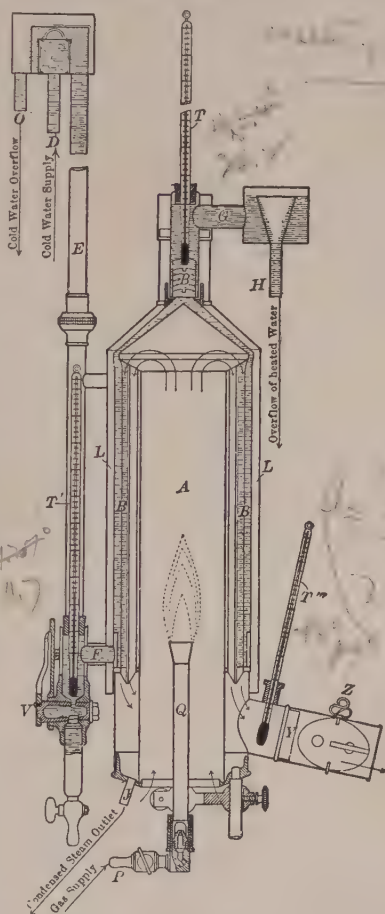


FIG. 25.

OBSERVATIONS AND CONCLUSIONS. Record all readings taken and arrange the data for the two determinations in parallel columns. From each test, calculate the heat of combustion of illuminating gas in calories per liter and in B.T.U. per cubic foot. Also reduce the first value to 0° C. and 76 cm. Hg. pressure.

EXPERIMENT 21

Dew-Point and Humidity of Atmosphere

OBJECT. To determine the hygrometric conditions of the atmosphere with the Daniell dew-point hygrometer and the wet- and dry-bulb psychrometer.

THEORY. The dew-point hygrometer consists of a vessel with an outer surface of blackened glass or polished metal, and containing a liquid which may be cooled in various ways below the dew-point. The temperature of the liquid at which the water vapor in the atmosphere begins to condense in minute drops of water on the glass or metal surface is the dew-point. Having measured the dew-point and the temperature in any given locality, the relative humidity of the atmosphere at that place may be obtained by computing the ratio of the known pressures (Table XII) of saturated aqueous vapor at these temperatures, since the pressure is proportional to the quantity of vapor present.

The psychrometer consists of two thermometers, one of which has its bulb surrounded by muslin that is kept saturated with water. When the instrument is placed in a current of air, the dry-bulb thermometer will indicate the temperature of the air, but the wet-bulb thermometer will indicate a lower temperature because of the evaporation of water from the muslin. The drier the air the more rapid will be the evaporation and therefore the greater the difference in the readings of the two thermometers. The depression of the wet-bulb thermometer indication depends to some extent upon the velocity of ventilation, but if the air in the vicinity of the bulb moves at a velocity

of 15 feet or more per second a constant maximum depression is attained. Under this condition the relative humidity up to temperatures of 140° F. may be computed by means of following formula, deduced by Professor Ferrel and employed as the basis of the Psychrometric Tables used by the U. S. Weather Bureau; namely, that the pressure of the aqueous vapor actually present at the temperature t° F. is

$$e = e' - 0.000367H(t - t') \left(1 + \frac{t' - 32}{1571} \right),$$

where e' is the maximum pressure of the vapor at the temperature t' , H is the pressure of the atmosphere in inches of Hg., and t and t' are the temperatures in degrees F. of the dry- and wet-bulb thermometers respectively. If f be the maximum vapor pressure at the temperature t of the dry-bulb thermometer, then the relative humidity, expressed as a percentage, is

$$h = \frac{100e}{f}.$$

In these expressions, the pressures e , e' and f are expressed in inches of Hg., and the latter two quantities are ascertainable from Table XII. To find the dew-point, refer to this table of maximum vapor pressures and find the temperature corresponding to the value of e .

The absolute humidity, or weight of aqueous vapor, in grains per cubic foot of the atmosphere at temperature t i.

$$m = \frac{352.4}{30} \cdot \frac{e}{1 + 0.00204(t - 32)},$$

where 352.4 is the density of water vapor in grains per cubic foot at 32° F. and at a pressure of 30 inches of Hg., and where 0.00204 is the coefficient of expansion of the vapor per degree F.

APPARATUS. Daniell dew-point hygrometer, ether, hygrodeik, barometer, and electric fan.

Dew-point Hygrometer. The Daniell dew-point hygrometer consists of two glass bulbs joined by a bent tube and mounted on a stand as shown in Fig. 26. The device contains ether, which is placed entirely in the lower bulb when a determination is made. The temperature of the ether is indicated by a thermometer enclosed in the lower bulb, and the temperature of the atmosphere

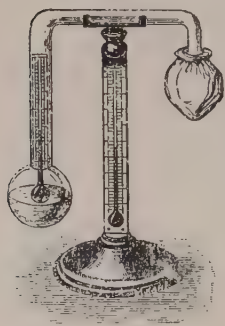


FIG. 26.

is given by the thermometer attached to the stand. The upper bulb is wrapped with a piece of muslin, and the lower bulb has a gilded rim on its surface. When ether is placed on the muslin, its evaporation cools the vapor within the bulb, some of the vapor condenses and consequently the vapor pressure is reduced. This reduction of pressure in the lower bulb causes the evaporation of some of the ether and this process cools the lower bulb. In this way the temperature

of the lower bulb gradually falls and the formation of dew on its outer surface may be observed. The appearance of moisture is best detected by moving a fine brush lightly across the gilded surface. The thermometers of the hygrometer used are graduated in degrees Réaumur; to convert to degrees F. multiply the reading by 2.25 and then add 32.

Hygrodeik. The hygrodeik is a psychrometer having a graphic chart over which a pointer is free to move. Two indexes are mechanically connected to the pointer, and when these indexes are adjusted along an auxiliary scale to indicate the readings of the two thermometers, then the position of the pointer on the chart shows directly the relative and the absolute humidity of the air as well as the dew-point.

PROCEDURE. Prior to using the Daniell hygrometer read both thermometers and derive the correction to be

applied to the exposed thermometer so as to agree with the other one. Then cool the ether in the lower bulb and note the temperature of the enclosed thermometer when dew appears. After cooling ceases and the temperature again rises, note the temperature of the ether when the moisture on the gilded surface disappears. The mean of these temperature readings is taken as the dew-point. Observe the temperature of the atmosphere by means of the thermometer on the hygrometer stand. Make a second determination.

Before wetting the muslin of the psychrometer thermometer, note the temperatures indicated by the two thermometers and deduce the correction to be applied to the dry-bulb thermometer in order to agree with the other. Then wet the muslin and place the hygrodeik in a current of air from the electric fan. When the wet-bulb thermometer indicates a steady reading, read both thermometers and the barometer. Set the indexes and pointer, and observe the humidity and dew point from the chart on the instrument.

Take the hygrodeik out-of-doors and make a similar determination of humidity and dew-point at the new location.

OBSERVATIONS AND CONCLUSIONS. Record all data taken in tabular form for each instrument. Reduce the thermometer readings on the dew-point hygrometer to degrees F., and find the pressure of saturated aqueous vapor from Table XII at the dew-point and at corrected room temperature and compute the relative humidity. From the corrected thermometer readings on the hygrodeik compute the relative humidity, dew-point, and absolute humidity and compare the results with the direct indications on the chart of the instrument.

EXPERIMENT 22

Thermal Conductivity of Metals

OBJECT. To measure the thermal conductivity of copper with Searle's apparatus.

THEORY. One end of a metal bar is continuously heated by steam, while the other end is kept cool by water which flows in a steady stream through a helical tube wound around that end of the bar. If the rod were perfectly insulated so that no heat could escape from the sides of the rod, then the heat which travels along the rod when conditions become steady is the same at every cross-section, and when it reaches the cooling tube it raises the temperature of the flowing water from t_0 to t_1 ° C. If m grams of the water are heated over this range every second, it follows that the heat traveling along the rod in calories per second will be

$$H = m(t_1 - t_0).$$

The temperature gradient along the rod may be measured by placing thermometers in suitable thermal contacts located at two places on the rod. Let the temperatures at these contacts be t_2 and t_3 ° C., and the distance between contacts be l cm. Then the temperature gradient in degrees C. per cm. is

$$G = \frac{t_3 - t_2}{l}.$$

The thermal conductivity of the rod of cross-sectional area A sq. cm. is

$$k = \frac{H}{AG}.$$

APPARATUS. Searle's conductivity apparatus, four thermometers, boiler, Bunsen burner, collecting vessel, graduate, and watch.

The apparatus used for measuring the thermal conductivity of a particular sample of metal is shown in Fig. 27. It consists of a cylindrical copper rod with a steam jacket at one end and a coil of several turns of small copper tube soldered around the other end of the rod. This tube terminates in two small copper vessels in which are held the bulbs of the thermometers which measure the temperature of the entering and outflowing water. The thermal contacts for measuring the temperature

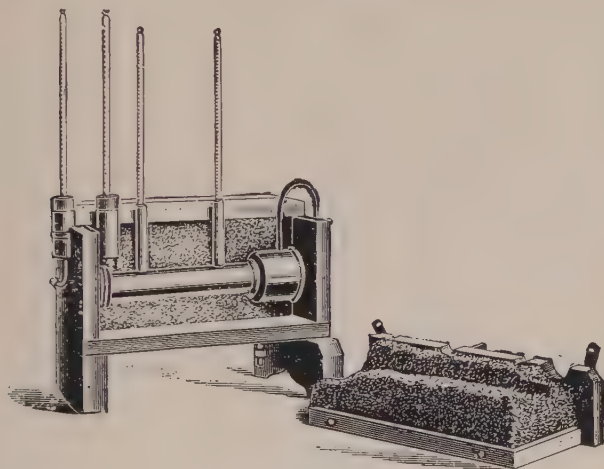


FIG. 27.

gradient are small hollowed copper cylinders which are brazed to the bar. Thermometers fit loosely into these contacts and the space around the thermometer bulbs is filled with oil in order to provide good thermal contacts. The whole bar with its fittings is covered with felt so that the escape of heat from the sides of the bar is minimized. The diameter of the rod is 3.81 cm. and the distance between thermal contacts is 10.16 cm.

PROCEDURE. Light the gas at the Bunsen burner beneath the boiler and when steam is formed pass it

through the steam jacket. Turn on the water through the cooling tube and regulate the rate of flow so that the temperature rise of the water will be at least 5°C . When each thermometer indicates a steady reading, collect the outflowing water in a vessel for a period of five minutes, taking readings of each thermometer at one-minute intervals during this time. Measure the amount of water collected with a graduate. Then change the rate of water flow through the cooling tube and take similar observations for another five-minute period. Thereafter turn off gas and water.

OBSERVATIONS AND CONCLUSIONS. Arrange the temperature readings in parallel columns and compute the average indication of each thermometer for each of the two tests. Thereafter calculate for each test the discharge of water in grams per second, the flow of heat in calories per second, the temperature gradient along the copper rod, and the thermal conductivity of copper. Compare the value of k obtained by experiment with the values given in Table XIII.

EXPERIMENT 23

Refractive Index of Prism

OBJECT. To measure the angle of a glass prism and to determine its index of refraction for sodium light by means of a spectrometer.

THEORY. Let a ray of light from a point source be reflected from one face of a prism to a definite point. Then move the prism through a measured angle so that a ray from the same source will be reflected from the second face of the prism to the same reference point. If the angle through which the prism was turned be θ degrees, then the angle of the prism between the two faces referred to will evidently be

$$\Phi = 180^{\circ} - \theta.$$

Consider a beam of monochromatic light incident upon one face of a prism at an angle i to its normal. The beam will enter the prism and make an angle r with this normal and will then pass out of the second face of the prism, making angles of incidence i' and refraction r' with the normal to the second face. From geometric considerations, the angle of the prism is

$$\Phi = r + i',$$

and the angle of deviation between the incident and emergent rays will be

$$D = i - r - i' + r'.$$

Since $\sin i = \mu \sin r$, and $\sin i' = \frac{1}{\mu} \sin r'$, where μ is the index of refraction of the prism (relative to air), it follows that

$$D = \sin^{-1}(\mu \sin r) - \Phi + \sin^{-1}(\mu \sin [\Phi - r]).$$

In order to find the relation between r and i' which will make the angle of deviation a minimum, differentiate the foregoing expression with respect to r and then equate the result to zero and solve. Thus

$$\frac{dD}{dr} = \frac{\mu \cos r}{\sqrt{1 - \mu^2 \sin^2 r}} - \frac{\mu \cos (\Phi - r)}{\sqrt{1 - \mu^2 \sin^2 (\Phi - r)}} = 0;$$

whence

$$\frac{\cos^2 r}{\cos^2 i'} = \frac{1 - \mu^2 \sin^2 r}{1 - \mu^2 \sin^2 i'},$$

and it follows that $r = i'$. Consequently the minimum deviation of the beam in passing through the prism occurs when the angles r and i' are equal, whereby angles i and r' are also equal. Then the angle of minimum deviation will be

$$D_m = 2(i - r),$$

and the refractive index of the transparent material forming the prism is

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin \frac{1}{2}(\Phi + D_m)}{\sin \frac{1}{2}\Phi}.$$

APPARATUS. Spectrometer, prism, combined fish-tail and Bunsen burner, and sodium bead on platinum wire supported on burner.

The spectrometer comprises an accurately graduated circular scale, a movable horizontal table, a collimator carrying a slit of adjustable width, and a telescope for viewing the slit, all mounted upon a base. The scale, table and telescope are constructed so that they may be moved about a common vertical axis, and are severally equipped with clamping screws for holding them in any desired position. The telescope is provided with cross-hairs, and its mounting carries a tangent screw for securing fine adjustment in locating the telescope, and a pair of verniers 180° apart for reading its position on the graduated circle. The instrument has been adjusted so that the optic axes of telescope and collimator are perpendicular to and intersect the vertical axis of the instrument. Before using the spectrometer for angular measurements, two adjustments must be made, namely: to focus the telescope, and to focus the collimator.

To focus the telescope, look through the telescope toward some bright object and adjust the position of the eye-piece so that the cross-hairs are in distinct focus. Then focus the telescope proper upon some distant object until its image lies in the plane of the cross-hairs as evidenced by lack of parallax, the eye-piece adjustment remaining unaltered.

To focus the collimator, place the telescope coaxial with the collimator, and place an illuminating flame back of the collimator slit. Look through the telescope and bring the image of the narrow slit into sharp focus and without parallax with the cross-hairs. This adjustment must be made by moving the slit toward or away

from the collimating lens while leaving the original adjustment of the telescope unaltered.

PROCEDURE. Carefully examine the graduations on the scale and verniers, and become familiar with the methods of coarse and fine adjustment of the spectrometer. Be sure to release the clamping screw of any part of the instrument that is to be shifted before attempting to move that part. Focus the telescope and collimator for parallel rays as already explained.

Angle of prism. To measure the angle of the prism clamp the telescope at an angle of approximately 60° with the collimator, and clamp the table to the graduated circle, which is left free to turn. Place the prism centrally on the spectrometer table and adjust the leveling screws on the table until the faces of the prism are vertical. Set the slit in a vertical position and illuminate it with the fish-tail burner. Then turn the table with prism so that the beam of light from the slit will be reflected from one face of the prism and enter the telescope, forming an image of the slit upon the vertical cross-hair. Determine this position of the prism by reading both verniers. Rotate the prism and scale until a similar reflection from the second face of the prism is obtained, and note its new position by reading both verniers. These observations provide the data for computing the refracting angle of the prism.

Angle of minimum deviation. To measure the angle of minimum deviation clamp the scale to the collimator, but release the knurled screw which clamps the spectrometer table to the graduated circle. Unclamp the telescope and move it into such position that, with the prism removed from the spectrometer table, the image of the slit will be received upon the vertical cross-hair of the telescope. Then read both verniers. Form a sodium bead at the bent end of a platinum wire which is supported in the hood of the burner. Adjust the hood so that the bead touches the Bunsen flame just above the burner, the burner being located about three inches back of the

collimator slit. Place the prism on the spectrometer table and rotate the telescope to receive the light from the collimator after being refracted through the prism. Turn the prism slowly and follow the image of the slit with the eye until the angle between the refracted ray and the prolonged collimator axis is a minimum. Then bring the telescope into position and watch the slit image in the telescope while slight movements of the prism are made. Adjust the prism to the position for which the deviation of the refracted ray is least and then move the telescope with the tangent screw so that its vertical cross-hair will bisect the image of the slit. After verifying this adjustment of the telescope by turning the prism slightly in each direction, take readings of both verniers. The data for obtaining D_m for sodium light are now at hand.

OBSERVATIONS AND CONCLUSIONS. Call one vernier *A* and the other *B*. Record all spectrometer readings in tabular form, placing the observations taken on the two verniers in parallel columns bearing the captions: Vernier *A*, and Vernier *B*. Compute the arithmetical mean value of θ and of D_m as given by the two verniers, in order to correct for eccentricity due to the non-coincidence of the axes of rotation of the graduated circle and the telescope. Determine the angle of the prism and then calculate the refractive index for sodium light of the glass of which the prism is constructed.

EXPERIMENT 24

Focal Lengths of Convex Lenses

Radius of Curvature of Concave Mirror

OBJECT. To determine the focal length of a thin converging lens by two methods, the focal length of a thick lens, and the radius of curvature of a concave spherical mirror by two methods.

THEORY. An object is placed before a thin converging lens at a distance away greater than the focal length of the lens. A real inverted image of this object will be formed on the other side of the lens, the usual construction lines being shown in Fig. 28. Let the distances from the center of the lens to the object of length O , to the image of length I , and to either focus F be called

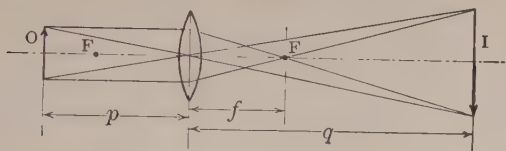


FIG. 28.

p , q and f respectively. Then from the geometry of the figure

$$\frac{O}{I} = \frac{p}{q} \quad \text{and} \quad \frac{O}{I} = \frac{f}{q - f}$$

whence

$$f = \frac{pq}{p + q}.$$

The focal length of a thin converging lens may be determined with the use of this equation in two ways: (1) by measuring the object distance p and the corresponding image distance q and substituting these values in the formula, and (2) by choosing an object at a great distance so that p may be considered infinite, and measuring the image distance q , which is then equal to f .

With a thick lens or a lens combination the object and image distances cannot be measured from its geometric center nor from the lens surfaces. However, the focal length of such lenses can be determined experimentally with sufficient accuracy by finding the distance between the two positions of the lens at which distinct images are obtained for a fixed distance between

object and image. If the lens must be moved a distance a from one of these positions to the other, and the distance between the object and image be l , then

$$a = q - p \quad \text{and} \quad l = q + p.$$

Substituting the values of p and q herefrom in the foregoing expression for f , there results

$$f = \frac{l^2 - a^2}{4l}.$$

The radius of curvature of a concave mirror can also be obtained from the equation $f = \frac{pq}{p + q}$ if it be remembered that parallel light incident near the vertex of such a mirror will be reflected to the focal point midway between the vertex and the center of curvature of the mirror. Thus, the radius of curvature $r = 2f$, whence

$$r = \frac{2pq}{p + q}.$$

Two methods of experimental procedure suggest themselves: (1) to measure the object distance p and the corresponding image distance q and substituting the values in the equation for r , and (2) to place the object in such position that the image will coincide with it, in which case $p = q$, and consequently $r = p = q$.

APPARATUS. Optical bench graduated in mm., thin converging lens, lens combination, concave mirror, plane mirror, white screen, straight-filament incandescent lamp, and mounted needle point.

The straight-filament incandescent electric lamp serves as the object in the measurements of focal length of lenses and in one of the methods employed for determining the radius of curvature of the concave mirror. The lamp,

lens and screen are mounted on separate carriages which may be moved along a 2-meter graduated optical bench.

PROCEDURE AND OBSERVATIONS. *Thin lens.* (1) Place the screen at distances varying from 130 to 200 cm. from the illuminated lamp at intervals of 10 cm., and for each setting adjust the lens at an intermediate position on the optical bench which will yield a distinct image of the object (lamp) on the screen. Note the distance from the lamp to the lens, p , and the distance from the lens to the screen, q , for each test, and place the readings in tabular form. Reserve one column in the table for calculated focal lengths.

(2) Replace the lamp by a mirror and adjust it to such an angle that a ray of sensibly parallel light from some distant object will be reflected onto the lens. Move either the lens or the screen until a sharply defined image of the distant object is projected on the screen. Measure the distance between the lens and screen. Repeat the adjustment and take another observation.

Lens combination. Interpose the lens combination between the illuminated lamp and the screen, and vary the distance from lamp to screen from 100 to 130 cm. in steps of 5 cm. For each setting locate the two positions of the lens on the bench which yield distinct images of the lamp filament on the screen, and note the distance between these positions. Record observations in tabular form, and reserve a column for the values of focal length to be calculated.

Concave mirror. (1) Place the mirror with its stand near the illuminated lamp and shift the screen until a well-defined image of the filament is obtained. The adjustment may be made more accurately if the mirror is partially covered so that only the portion near its vertex is exposed. Measure the distances from the mirror to the lamp and to the screen. Then make another independent setting and record results.

(2) Set the mounted needle point as object before the mirror and place the eye in the prolonged line through

the needle point and vertex of mirror. Adjust the position of the needle along the line of vision until no parallax between object and image is evident when moving the eye to either side; the needle is then at the center of curvature of the mirror. Measure the distance from the vertex of the mirror to the needle point. Repeat the adjustment, and take another observation.

CONCLUSIONS. Compute from the observations taken the average focal length of the thin lens by both methods, the average focal length of the lens combination, and the average radius of curvature of the concave mirror by both methods, all results being expressed in cm.

EXPERIMENT 25

Calibration of Ocular Scale of Cathetometer

OBJECT. To calibrate the eye-piece scale of a cathetometer microscope with a standard scale, and to determine the ratio of the centimeter to the inch.

THEORY. In this experiment the eye-piece of the cathetometer microscope is adjusted to yield an image of the transparent ocular scale at the distance of distinct vision, and the objective is adjusted to produce an image (with the aid of the eye-piece) of the standard scale at the same distance, so that both scales will be seen through the microscope superimposed upon each other. By observing the number of divisions on the ocular scale which is subtended by some integral number of divisions on the standard scale, the calibration of the ocular scale is effected. If S ocular divisions are subtended by L cm. on the standard scale, the value of an ocular division will be $\frac{L}{S}$ cm.

APPARATUS. Cathetometer with ocular scale, standard steel cm. and inch scales, and holder for standard scale.

The cathetometer consists of a horizontal microscope mounted on a vertical stand so that its axis may be raised or lowered. The height of an object can be measured accurately by the cathetometer by focusing the microscope successively at the top and the bottom of the object and noting the difference of the elevations of the microscope in the two positions by means of a vernier moving along a vertical scale. Very small vertical distances may be measured by means of the ocular scale without moving the microscope, after this scale has been calibrated.

PROCEDURE. Mount the standard centimeter scale in a vertical position and place the cathetometer a short distance away and level its microscope. Remove the eye-piece of the microscope and focus it so the divisions on the image of the ocular scale will appear clear and distinct. Replace the eye-piece with its scale vertical and then adjust the length of the microscope draw tube to division 13 on the tube scale. Then focus the microscope on the standard scale by moving the entire microscope back and forth by means of its twin knurled screws. A correct focus is indicated by the absence of parallax, that is, the lack of relative movements of the images of the two scales when the eye is moved vertically before the eye-piece. Determine the number of divisions on the ocular scale that is subtended by the largest integral number of divisions of the standard scale visible over the range of the ocular scale. Increase the length of the tube in steps of 1 cm. up to its limiting length, and at each setting take a reading as before.

Replace the cm. scale by a standard inch scale and make another set of observations in the manner described.

OBSERVATIONS AND CONCLUSIONS. Evaluate the length of a division of the ocular scale for each tube setting, both in cm. and in inches. Put these values together with the observations in tabular form. Plot a curve with tube lengths as abscissas and with the

values of an ocular division (that is, L/S) in cm. as ordinates. For each tube length find the ratio of the values of L/S determined on the inch and cm. standard scales. The average value of this ratio gives the number of cm. in one inch.

EXPERIMENT 26

Curvature of Cornea of Eye with Ophthalmometer

OBJECT. The measurement of the radii of curvature of the anterior surface of the cornea at various axes and the determination of corneal astigmatism of the human eye by means of the Javal-Schiötz ophthalmometer.

THEORY. Astigmatism is the most common of the refractive errors of the human eye, and is due almost entirely to asymmetry of the cornea. The curvature of the cornea of an astigmatic eye has different values at various meridians, the two meridians giving the maximum and minimum refraction at the corneal surface being termed the chief meridians. These meridians are at right angles to each other, but are not necessarily horizontal and vertical respectively. The vertical (or more nearly vertical) chief meridian of the cornea has usually the maximum and the other meridian the minimum of curvature; when this is the case, it is spoken of as *astigmatism with the rule*, and when the reverse is true it is called *astigmatism against the rule*.

The ophthalmometer is an instrument for observing the positions of the images of two illuminated objects formed by reflection from the surface of the cornea, which acts as a convex mirror. The eye to be observed is placed at the center of a horizontal arc along which the luminous objects or mires may be moved so as to alter the distance between them. This arc is rotatable about a horizontal axis coincident with the optic axis of the eye in order to bring the mires into any corneal meridian. If the distance between the mires be determined as well as the distance between their images

formed by the cornea, then the corneal curvature may be computed.

In Fig. 29, EE is the anterior surface of the cornea, so placed that it is concentric with the arc AHB , upon which are supported the mires M and N . The inner edges of these mires will be taken as the reference points and the distance between the edges, AB , will be considered as the size of the object. Three rays from A are incident

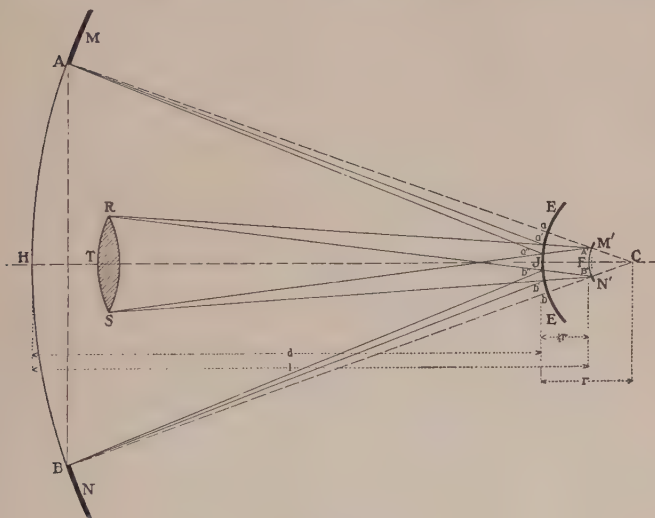


FIG. 29.

upon the cornea at the points a , a' and a'' , where they are reflected to A , R and S respectively. Likewise three rays from B are incident upon the cornea at b , b' and b'' , where they are reflected to B , S and R respectively. The two groups of reflected rays appear to come from the points A' and B' back of the reflecting surface, and these points are the inner edges of the virtual images M' and N' of the two mires. The distance between A' and B' will be the size of the image.

Fig. 29 is not drawn to scale, but some parts are

magnified and others reduced in order to attain clearness. In the ophthalmometer the radius CH of the arc AB is about 35 times as large as the radius r of the cornea. Consequently the object AB may be considered very remote and the image $A'B'$ will practically be at the principal focus F of the cornea, which lies midway between its vertex J and center of curvature C . If the sizes of object and image be respectively O and I , and if their distances from the cornea be respectively d and $\frac{r}{2}$, then

$$\frac{O}{I} = \frac{d+r}{\frac{r}{2}} = \frac{2l+r}{r},$$

where l is the distance between object and image; whence the radius of curvature of the cornea is

$$r = \frac{2lI}{O-I}.$$



FIG. 30.

The measurement of the size of the image I is accomplished by viewing the images of the mires by a telescope containing a Wollaston bi-refringent quartz prism, and shown diagrammatically in Fig. 30. The prism B , placed between the similar achromatic lenses A and C , is a rectangular parallelopiped consisting of two similarly shaped triangular prisms, so cut that in one the axis of the crystal is perpendicular to the apex of the triangle and in the other parallel thereto. The rays from each mire, after reflection from the cornea, enter the prism where they are separated into two sets a and

b, the extraordinary and ordinary rays. These rays are brought to a focus and two images are formed of each mire at the focal plane D , where the images are viewed by an eye-piece. Since the lenses A and C have the same focal length, and since lens A is oriented so that its principal focal plane coincides with the image back of the cornea ($A'B'$ of Fig. 29) the size of this image will be reproduced by lens C in the focal plane at D . The separation of the two images of each mire by the prism, or the amount of doubling, in the Javal-Schiötz ophthalmometer is 3 mm., and remains constant irrespective of the separation of the mires. In use, the mires of this ophthalmometer are shifted until the ordinary image of one mire, as seen through the telescope, touches the extraordinary image of the other mire, in which case the size of the image $I=0.3$ cm. Then, since the distance l in this instrument is 27 cm., the radius of curvature of the cornea under examination is given directly as

$$r = \frac{2 \times 27 \times 0.3}{0 - 0.3} = \frac{16.2}{0 - 0.3} \quad \dots \quad (1)$$

Thus, if the mires were separated 20.5 cm., in order to have their images touch for a certain eye, then the radius of curvature of its cornea would be $\frac{16.2}{(20.5 - 0.3)}$ or 0.8 cm.

APPARATUS. Javal-Schiötz ophthalmometer, indicating lens tester, and cm. scale.

The construction of the ophthalmometer is indicated in Fig. 31. It consists of a head-rest with an adjustable chin-piece, of two mires which can be moved apart along a circular arc by means of spiral tracks upon the front face of a dial, of a telescope containing the bi-refrangent prism and lenses and equipped with rack and pinion as well as an elevating arrangement for bringing either eye of any subject into focus, and of a scale on the rear face of the dial for indicating directly by the

use of suitable pointers the inclination of the mire arc, the radius of curvature of the cornea, and its astigmatism expressed in diopters. The mires are illuminated by miniature electric lamps and are of dissimilar shape, one is a rectangle and the other is a figure consisting of steps, both mires being divided into halves by black lines. When the rays from the mires after reflection from the cornea are viewed through the telescope, two images of each mire will be observed and will appear as in Fig. 32 for the horizontal and vertical positions

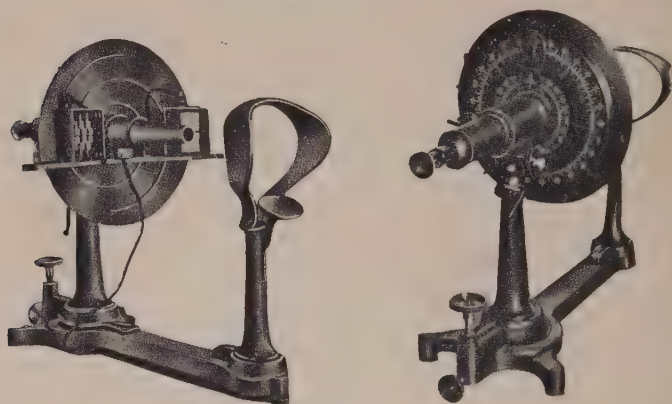


FIG. 31.

of the mire arc, P and Q representing the ordinary and P' and Q' the extraordinary images. Only the two central images, namely P' and Q , are considered in making a determination of corneal astigmatism.

Fig. 32 for the horizontal position shows that the mires were so moved that the inner edges of the images P' and Q touch, indicating that the size of the image I ($A'B'$ in Fig. 29) is 0.3 cm. Assuming that the cornea is astigmatic with the rule, then, when the mire arc is turned 90 degrees to the position shown at the right of Fig. 32, it will be found that the images P' and Q overlap, the white portion of image Q representing the

amount of overlap. In this case the radius in the vertical meridian is less than in the horizontal, and therefore the size of image I is decreased. In order to make $I=0.3$ cm. in the vertical position it is necessary to move the mires until the images P' and Q just touch. The radius of the cornea in either meridian is then readily determined from the corresponding separation O of the inner mire edges by means of Equation (1). The

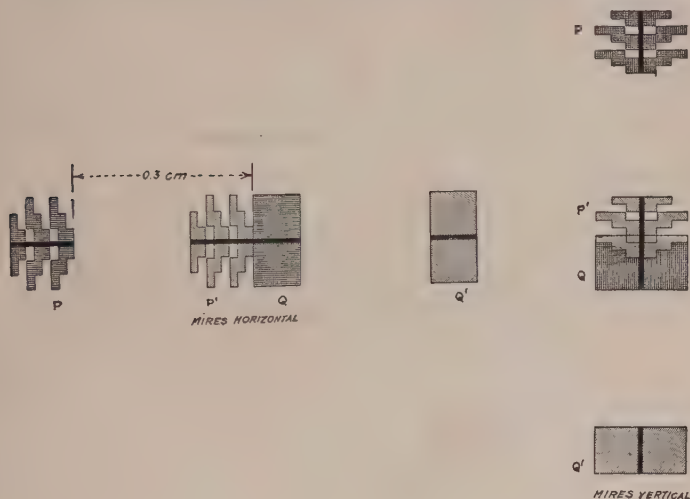


FIG. 32.

rear of the ophthalmometer dial is graduated in accordance with this equation so that when the disk is turned in the process of moving the mires until their images touch, the radius of curvature of the cornea will be indicated directly in mm. against an index.

The principal focal distance, f cm., of a lens constructed of material having a refractive index μ and having radii of curvature r and r_1 cm. is given by the usual lens formula

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{r} - \frac{1}{r_1} \right).$$

For a plano-convex lens r_1 becomes infinity, and therefore

$$f = \frac{r}{\mu - 1}.$$

Considering the cornea of radius r to be merely the transparent shell of the aqueous humor of refractive index 1.337 (see Table XV), the refractory power of the cornea expressed in diopters is

$$D = \frac{100}{f} = 100 \frac{\mu - 1}{r} = 100 \frac{1.337 - 1}{r} = \frac{33.7}{r}. \quad (2)$$

Thus, when the radius of curvature of the cornea in the horizontal meridian is 7.5 mm., and in the vertical meridian is 7.0 mm., then the refractory power of the cornea in the two meridians is $33.7 \div 0.75 = 45$ diopters and $33.7 \div 0.7 = 48.1$ diopters, and the astigmatism is $48.1 - 45.0 = 3.1$ diopters "with the rule." The dial of the ophthalmometer also carries a dioptic scale graduated in accordance with Equation (2), so that the refractory power of the cornea can be read directly at either axis.

PROCEDURE. Seat the individual whose eyes are to be tested in position so that the upper part of the head-rest is just above his brows, adjusting the chin-rest by means of the horizontal milled screw at the rear of the ophthalmometer. Obstruct one eye with the shutter and by looking through the sight holes in the dial align the telescope with the other eye. Illuminate the mires. Adjust the eye-piece of the telescope by turning it so that the cross-hairs may be viewed free from parallax. Find the mire images in the telescope by raising or lowering it with the use of the vertical milled screw and by swinging it to right or left, as required, in order to bring the two central images at the intersection of the cross-hairs. Focus the telescope by means of the milled wheel at its side until the images are seen distinctly; do not alter this adjustment throughout

the measurement of this eye. Locate the primary axis by grasping the telescope at the leather-covered tube and turning it, not more than 45 degrees either side of the zero mark, until the continuity of the two central black lines through the mires is established. Then bring the mires together until their edges just touch by revolving the dial with the aid of the capstan pins. Finally move the pointer with the extension hook directly back of the other pointer so as to be covered by it.

Revolve the telescope through the zero mark to the secondary position which is just 90 degrees from the other chief meridian, and it will be observed that the black lines through the mires again lie in one line. If the eye under examination is astigmatic, the mire images will appear overlapped or separated. Then adjust the mires by turning the dial so that their images again touch. It will now be observed that the two pointers occupy different positions on the dioptic scale, and the difference between them is the astigmatism in diopters, and the axes of the two chief meridians are indicated on the small disk.

OBSERVATIONS AND CONCLUSIONS. Verify the accuracy of the scales on the ophthalmometer dial by measuring the distance O between the inner edges of the mires and using Equations (1) and (2). Test the right eye (O. D.) and the left eye (O. S.) of one member of the group performing this experiment. Record these readings and add those taken on your own eyes. ~~If you wear glasses measure the curvature and cylindricity of each spectacle lens by means of the indicating lens tester.~~ Place the observations on the four eyes in tabular form and indicate the names of the individuals concerned in the tests. Determine the amount, the principal meridians and the nature of the astigmatism of your eyes and state the extent to which your glasses remedy this defect of vision.

EXPERIMENT 27

Magnifying Power of a Compound Microscope

OBJECT. To determine the magnifying power of a compound microscope by calculations based upon its tube length and upon the trade markings on its objective and eye-piece, and to verify this result by direct experiment.

THEORY. The optical elements of a compound microscope are an objective, made up of several lenses and having a short focal length, and an eye-piece, generally made of two lenses and also having a short focal length. A strongly illuminated object, often in the form of a slide, is placed just a little distance beyond the principal focus of the objective. The objective forms a real magnified image of the object in space on the other side and some one or two hundred millimeters away from the objective. This magnified real image is viewed by an observer through the eye-piece just as though he were using an ordinary simple magnifying glass. The image which he sees is at the distance of distinct vision, usually 250 millimeters.

To determine the magnification produced by the objective, that is, the ratio of the distance between two points of the image to the distance between the corresponding points of the object, one may rely upon the relations expressed by the ordinary lens formula:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}, \quad \text{ (Exp. 24)}$$

wherein p and q represent respectively the distances of the object and image from the lens and f is the principal focal length of the lens. In order to make use of this formula, consideration must be given to the great axial thickness of the lens combination which constitutes the objective. On the axis of a thick lens there are two particular points which are termed the first and second principal points, and these points are usually close

together. Were an object placed in a plane passed perpendicular to the axis through the first principal point, an image would be formed in a similar plane passed through the second point and it would be of the same size as the object, consequently the magnifying power will be unity. If, now, the distance of some remote object be measured from the first principal point and the distance of the resulting image from the second principal point, then the ordinary lens formula just given applies as well to thick as to thin lenses. The principal focal length must similarly be measured from the focus of parallel rays to the principal point on the same side of the lens as this focus. When so measured the two principal focal lengths on opposite sides are numerically equal to each other. This length is usually indicated upon the mountings of microscope objectives.

Since, in using the microscope the object is placed just slightly beyond the principal focus, the object distance may be considered as equal to the focal length f . In order to cut off extraneous light, the rays which extend from the objective to the magnified real image of the object are enclosed in a light-tight tube and the distance T from the image to the second principal point is termed the optical tube length. The corresponding linear dimensions of the image and object will vary directly as their distances from their respective principal points. The magnification produced by the objective may therefore be taken as

$$m_o = \frac{T}{f}.$$

The magnification produced by the eye-piece of focal length f' millimeters is in accordance with the usual expression:

$$m_e = \frac{250}{f'}.$$

When the eye-piece is used for viewing the image produced by the objective, the total magnification is equal to the product of m_o and m_e , or

$$m = m_o m_e = \frac{250T_1}{ff'} \cdot 140 \quad (1)$$

all dimensions being in millimeters.

Eye-pieces usually consist of two lenses, the magnifying lens in the end toward the observer's eye, and a field or collective lens which enables the observer to view a large field and to secure simultaneous focus of the central and peripheral portions of the field. They are usually stamped with a numeral indicating their magnifying powers used simply as a magnifier and often with equivalent focal length, that is, the focal length of a thin lens which would have the same magnifying power. Within these eye-pieces is a diaphragm so located that when the microscope is focused the real image produced by the objective and aided by the field lens will be formed at this diaphragm. The optical tube length is then the distance between this diaphragm and the second principal point of the objective. It is the present practice to have this latter point located in the plane of the flange of the mounting of the objective, as at A, Fig. 33 (this plane passes through the upper point of the letter A). The equivalent focal length of eye-pieces for magnifying powers of 5, 10 and 15 are respectively 50, 25 and 16.7 millimeters.

If the magnifying powers m_1 and m_2 of a microscope with two unknown optical tube lengths T_1 and T_2 , whose difference, however, may be measured on the draw tube scale, be experimentally determined, then, since, $m_1 = \frac{250T_1}{ff'}$, and $m_2 = \frac{250T_2}{ff'}$, it follows that

$$ff' = \frac{250(T_1 - T_2)}{m_1 - m_2} \quad (2)$$

Substituting this value of ff' in the equations for m_1 and m_2 yields

$$T_1 = \frac{m_1(T_1 - T_2)}{m_1 - m_2}, \quad \text{and} \quad T_2 = \frac{m_2(T_1 - T_2)}{m_1 - m_2}.$$

APPARATUS. A compound microscope with one objective and one eye-piece, a stage micrometer slide ruled to 0.01 mm., a millimeter scale, an illuminating arc-lamp with rheostat, a drawing board with light shield and attached stand for holding the microscope.

The microscope, Fig. 33, consists of a base B , pillar P , stage S , and handle-arm H , to the last of which is attached by a rack and pinion the microscope tube T . Within this tube there is movably fitted the graduated draw tube D , in whose upper end is placed the eye-piece E . To the lower end of the body tube is fastened a rotatable nose piece N into which is screwed the objective O . Rays of light from the source of illumination

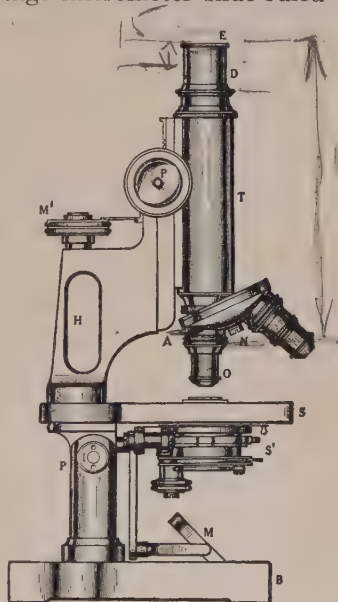


FIG. 33.

are reflected by the universally movable mirror M , through the condenser in the substage S' upon the stage micrometer slide placed directly under the objective. In use, the upper portion of the microscope with the stage will be turned through 90 degrees around the inclination joint I , and will then be mounted upon the extension of the drawing board as shown in Fig. 34. The arc light connected in series through the rheo-

stat to the electrical supply mains is adjusted to send light upon the mirror *M*, which redirects it through the tube. When properly focused a real image will be formed upon a sheet of paper lying on the drawing board. Fig. 35 shows the microscope fitted with a micrometer eye-piece for measuring distances on the object viewed by the microscope.

PROCEDURE. Connect the arc light in series with the rheostat to the 110-volt service mains, the carbons being originally separated. Then turn the regulating

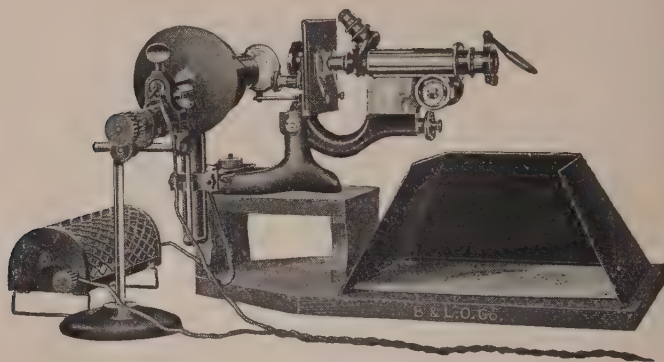


FIG. 34.

screws of the lamp until the carbons just make contact and then *immediately* separate the carbons a short distance (if the carbons be left in contact, the rheostat is likely to become injured). Direct the rays of the lamp upon the microscope mirror at right angles to the axis of the microscope tube, which has been mounted as shown in Fig. 34. Then adjust the mirror so as to send the reflected rays from the lamp through the tube. Adjust the tube length so that the distance from the flange of the objective mounting to the eye-piece end of the draw tube is 160 mm. Place a sheet of paper upon the drawing board and focus the stage micrometer image



FIG. 35.

upon the paper. In focusing the microscope the tube should be at first lowered carefully until the objective just clears the object, and then, while looking through the instrument, the tube should be gradually *withdrawn* by turning the pinion head *P* (Fig. 33) until the object becomes visible, after which the micrometer head *M'* should be used to obtain a sharp focus. Great care should be taken with objectives, which are made of soft glass, so as not to scratch their surfaces by coming in contact with the object observed.

Mark upon the paper two lines corresponding to two divisions of the image which are separated by 100 divisions. The distance between these marks corresponds in the image to one millimeter in the object. Measure this distance in millimeters, and the result is the magnifying power for this tube length. Then increase the tube length by a measured number of millimeters, say 40 or 50, which distance can be read off the scale on the draw tube. Then repeat the process and determine the new magnifying power. Finally remove the eye-piece and, after unscrewing the eye lens, measure the distance in millimeters from the diaphragm to the upper edge of the eye-piece mounting.

OBSERVATIONS AND CONCLUSIONS. Record all measurements in tabular form, and also note the markings of equivalent focal length on objective and eye-piece. From these markings and from the measured optical tube length compute the magnifying power of the microscope from Equation (1). Compare this value with the magnifying power as observed by test for the same tube length. Then calculate from the measurements of magnifying power for two different tube lengths the value of the product of the focal lengths of objective and eye-piece by using Equation (2), and compare this product with that resulting from the maker's ratings on these parts. (Manufacturers of microscopes guarantee focal length ratings to be accurate to 2 per cent.) Also calculate the values of optical tube lengths for the two settings of the draw

tube and compare them with the distances between the eye-piece diaphragm and the flange of the objective mounting for the same settings.

EXPERIMENT 28

Wave-lengths of Light by Interferometer

OBJECT. To measure the wave length of sodium light by means of the Michelson interferometer.

THEORY. The Michelson interferometer consists of two parallel-sided glass plates of equal thickness, shown

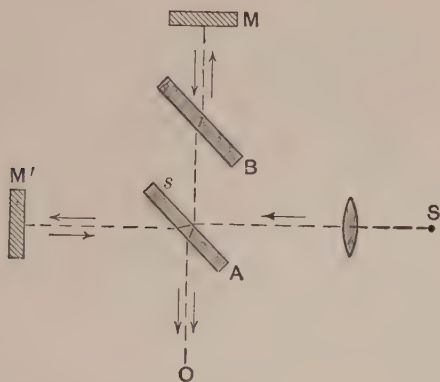


FIG. 36.

at *A* and *B* of Fig. 36, and two plane mirrors, shown at *M* and *M'*. The glass plate *A* is half-silvered on the face marked *s*, so that an incident beam of light from a luminous source *S* will be divided into two beams, one of which is reflected to mirror *M* and the other is transmitted to mirror *M'*. After reflection at their respective mirrors, these beams are returned along their initial path and pass through plate *A* to the observer at *O*. The clear glass plate *B* is introduced between

A and M so that both beams will pass through the same thickness of glass. When the two mirrors are exactly perpendicular to the beams of monochromatic light incident thereon, and when the mirrors are located at precisely equal distances from the half-silvered plate, then the two beams upon reunion at O are in condition to interfere, because of their opposition in phase occasioned by the dissimilar reflections at A . Should mirror M' be moved perpendicularly to itself a distance of one-half wave-length, the reunited beams will again

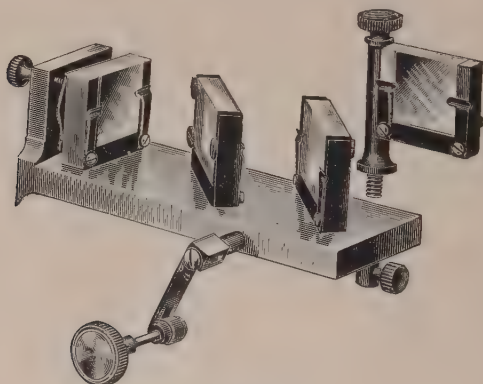


FIG. 37.

interfere, since the path from A to M' and back to A has been increased by a full wave-length. At such interferences the field of view at O will appear dark, while midway between them the field will be illuminated. If mirror M' be moved slowly in one direction over a distance l cm., while the number n of darkenings of the field be counted, then the wave-length of the monochromatic light used will be

$$\lambda = \frac{2l}{n} \text{ cm.}$$

APPARATUS. Interferometer attached to a micrometer slide which is mounted in a heavy adjustable support, sodium burner, and observing telescope.

The interferometer is illustrated in Fig. 37, which shows the two glass plates firmly mounted on a metal plate. The fixed mirror is mounted at one end of this plate and is equipped with fine adjustment screws; the movable mirror is shown unattached. The metal plate is clamped across the bed plate of the micrometer slide, which is shown in Fig. 38. The micrometer screw of this slide has a pitch of 0.05 cm., and carries a head divided into 100 parts; thus movements of the carriage,

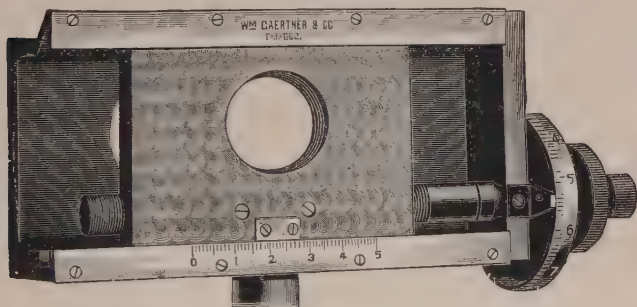


FIG. 38.

upon which is mounted the movable mirror (M' of Fig. 36), can be read to 0.00005 by estimating tenths of divisions on the micrometer head. A pinion shown at the bottom of Fig. 37 can be adjusted to engage the finely knurled edge of the micrometer head of Fig. 38, so that the carriage may be given a slow motion.

PROCEDURE. *Do not touch any optical surfaces of the apparatus.* See that the interferometer attachment is firmly clamped to the bed plate of the micrometer slide. Then move the slide until the mirror on it is at a distance away from some reference point on the half-silvered surface s of plate A equal to the distance of the fixed

mirror from that same point. This adjustment may be effected by means of dividers or with the aid of the mm. scale on the micrometer slide. Place the sodium burner near the principal focus of a short-focus lens and allow the resulting parallel beam of yellow light to fall upon plate *A*. While looking into the instrument from *O*, Fig. 36, shift the lens or burner to such position as will yield uniform illumination of the entire surface of plate *A*. Now hold a pin between this plate and the lens and three images will be observed, the third image being introduced by reflection from plate *A* at its unsilvered side. To eliminate the odd image, alter the inclination of mirror *M* by its adjusting screws until a coincidence is established between this image and either one of the other two.

When properly adjusted, there will be visible from *O*, across mirror *M*, a series of bands, called interference fringes, which represent the intersection of that mirror with a series of interference planes. These fringes are to be found by moving the eye sidewise and to and from plate *A*. If not directly visible, carefully adjust the screws at mirror *M* and search for the fringes. When they have been located, again adjust the position of mirror *M* so that the fringes will be vertical and that about a half dozen will be simultaneously in view. As the micrometer slide is moved slowly in either direction, these fringes will progress across the field of view, a movement of one fringe to the next corresponding to an advance on the part of the movable mirror of one-half wave-length of the light under investigation.

OBSERVATIONS AND CONCLUSIONS. Focus the observing telescope on the fringes and note the position of the movable mirror by reading the mm. scale and the divisions on the micrometer head. Turn the micrometer screw slowly by means of the pinion head until 200 fringes have passed the vertical cross-hair of the telescope and again note the position of the movable mirror. Repeat this process until 1000 fringes in all have been counted.

If l cm. be the distance moved by the carriage supporting the movable mirror for the passage of 1000 fringes, then the wave-length of yellow light is $\lambda = l/500$ cm. Compare this result with the value given in Table XIV.

EXPERIMENT 29

Wave-lengths of Light by Diffraction

OBJECT. To measure the wave-length of several colored lights by means of a reflection diffraction grating.

THEORY. The reflection grating consists of a large number of parallel reflecting metal strips separated by narrow non-reflecting spaces. The action of such a grating is illustrated in Fig. 39, in which the lines a , b , c , d and e represent horizontal sections of successive vertical reflecting strips, I shows an incident plane wave-front of monochromatic light striking the plane of the grating at an angle i , D represents the first-order diffracted wave-front, making an angle δ with the grating, and s is the distance between successive centers of the reflecting strips. For the first-order diffracted wave-front shown at D , the distance $p + q$ is one wave-length or λ , but in general, for the n th-order diffracted wave-front

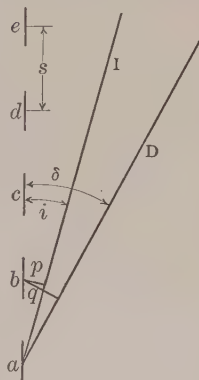


FIG. 39.

$$p + q = n\lambda.$$

Consequently from the figure

$$s \sin i + s \sin \delta = n\lambda,$$

whence the wave-length is

$$\lambda = \frac{s}{n} (\sin i + \sin \delta).$$

When the incident beam is normal to the surface of the grating it follows that $\sin i = 0$ in the expression for λ . Further, $\sin \delta$ may be expressed in terms of l and m ; consequently the wave-length of that color whose fringe is m cm. from the zero mark of the scale, is

$$\lambda = \frac{ms}{n\sqrt{l^2 + m^2}}.$$

APPARATUS. Speculum-metal diffraction grating, arc or incandescent-lamp projection lantern with optical bench, slit of adjustable width, and two cm. scales.

The grating to be used has 14,438 reflecting strips per inch, therefore the distance between successive strips is 0.00017592 cm. Do not touch the surface of the grating.

PROCEDURE. Place the arc light or the concentrated-filament lamp at the principal focus of the condensing lens of the projection lantern. Clamp the slit on the optical bench near the condensing lens, and place the projection lens some distance in front of the slit. Place the grating in front of the projection lens so that the reflecting strips are very slightly inclined to the vertical and that the normal to the grating surface and the lens axis lie in the same vertical plane. Mount the straight scale in a horizontal position just above and perpendicular to the axis of the projecting lens, with the zero mark of the scale vertically above this axis. Illuminate the arc or lamp, and adjust the positions of the grating, lens and scale until a clearly defined white image of the slit is formed by reflection at the zero mark of the scale. If the slit be made quite narrow, the diffracted rays will be seen as continuous spectra on both sides of the reflected image.

OBSERVATIONS. When the parts are in proper adjustment, note the distance m of the violet, green, yellow and red fringes from the zero mark of the scale and measure the perpendicular distance l of the grating from the scale. If the scale extends only to one side of the lens

the grating may be turned 180° around the normal to its surface in order to take observations of the other spectral image. Readjust the apparatus and take another set of readings.

CONCLUSIONS. Compute the wave-lengths of the colors mentioned above from readings on the first-order spectral images. Express the results also in microns (1 micron = 0.001 mm. = μ). Refer to Table XIV.

EXPERIMENT 30

Photometric Test of Incandescent Lamp

OBJECT. To determine the candle-power of a tungsten-filament lamp at various angles in a plane through the tip and base of the lamp with the Lummer-Brodhun contrast photometer. The results are plotted in polar coördinates to form a light-distribution curve, from which the mean candle-power in that plane may be determined.

THEORY. The lamp to be tested and the standard lamp are clamped at opposite ends of an optical bench and between them is a movable opaque white screen, both sides of which may be viewed simultaneously by an optical arrangement. The position of the screen is altered until the illumination of both sides of the screen is identical. If, after this adjustment, the distance of the lamp under test of candle-power I , from the screen be l , and the distance of the standard lamp of candle-power I_s from the screen be l_s , then since the intensity of illumination varies inversely as the square of the distance from the light source, it follows that the illumination of the photometer screen is

$$E = \frac{I}{l^2} = \frac{I_s}{l_s^2}.$$

53.5

.000624 cm.

53.5 | 5.033420
 22120
 1320

.00017592

17592
 17592
 .00017592

As the distance L between the lamps remains unchanged, the candle-power of the lamp under test may be expressed as

$$I = I_s \left(\frac{L - l_s}{l_s} \right)^2.$$

If the candle-power of a lamp be measured at various angles in a plane through its tip and base, these values when plotted on polar coördinate paper would form the light-distribution curve of the lamp in that plane. Had the lamp been rotated about its own axis during the candle-power measurements the resulting distribution curve would be the average for every plane containing the lamp axis. A distribution curve shows that the intensity of illumination from a lamp is not the same in all directions, therefore for purposes of comparison it is desirable to determine the mean spherical candle-power of the lamp. This cannot be obtained by merely averaging the candle-powers of the lamp as measured at equal angular intervals because of the wide difference in the corresponding zonal areas. It has been computed that the angles *measured from the lamp axis* at which values may be read from a light-distribution curve and directly averaged in order to obtain the mean spherical candle-power are: 24, 41, 54, 65, 75, 85, 95, 105, 115, 126, 139, and 156 degrees (see *Standard Handbook for Electrical Engineers*, ed. 4, p. 1138).

APPARATUS. Two-meter photometer bench with three carriages for supporting screen and lamps, Lummer-Brodhun photometer head, standardized tungsten filament lamp, lamp to be tested, slide-wire rheostat, and voltmeter.

The Lummer-Brodhun photometer head is illustrated in Fig. 40.

The screen S has its two sides illuminated by the light sources L_1 and L_2 , and these surfaces may be viewed simultaneously by means of the mirrors M_1 and M_2 , the right-angled prisms P_1 and P_2 , and the telescope. The

prisms are ground so that the actual surface of contact between the two prisms has the shape indicated by the unshaded part of F , while the shaded part shows where a narrow layer of air separates the prisms. Light from the right side of screen S after reflection from mirror M_1 enters prism P_1 normally, and that part which strikes the junction surface in the places where the two prisms are not in contact is totally reflected out of the prism and

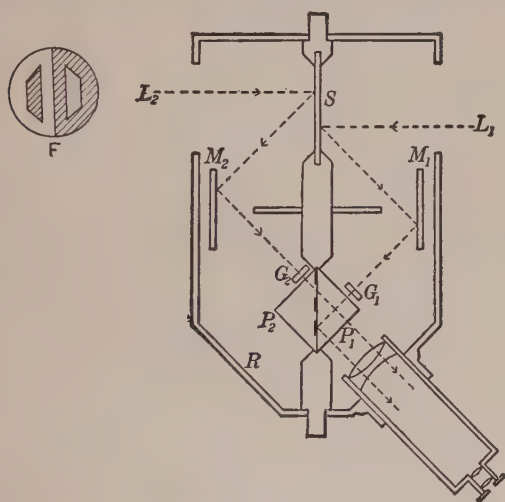


FIG. 40.

into the telescope, while the remaining portion of the entering light strikes the contact surface and is transmitted directly through prism P_2 to R . Similarly a part of the light from the other side of the screen is transmitted to the telescope while the remainder is totally reflected to R . The cross-sections of the beams of light entering the telescope will then resemble the configuration shown at F , wherein the shaded parts depict the cross-sections of the beams from the right side of S , and the unshaded parts show the beams from the left side of the

screen. Thus each of the two light sources illuminates a trapezoidal-shaped figure within a semicircular field that is illuminated by the other source. The rays of light from the two sides of the screen which illuminate the trapezoids are shown by the dotted lines in the figure. Both trapezoids are made a few per cent darker than their corresponding opposite semicircular fields by the interposition of the thin glass plates G_1 and G_2 , which absorb a small amount of the light going to the trapezoids.

To take a reading with the Lummer-Brodhun photometer, the screen is moved to such a position between the two light sources that the two trapezoids are both darker than the surrounding semicircular fields and the contrasts between each trapezoid and its surrounding field are identical. When the two sources are of exactly the same color the dividing line between the two semicircular fields will disappear when the screen is properly located.

PROCEDURE AND OBSERVATIONS. Place the standard lamp in its socket at the center of the carriage table so that the mark etched on its bulb is directed toward the photometer screen. Place the lamp to be tested in the socket on the swivel arm of the other carriage, and turn the arm till its associated graduated disk indicates 0 degrees against its stationary index. Move the two lamp carriages to the ends of the photometer bench and clamp the standard lamp at the zero mark and the lamp under test at the 200 mark of the cm. scale engraved on the photometer bench. Adjust the heights of the lamps and the screen so that the centers of the filaments and of the screen lie in one horizontal line. The lamps are connected in parallel to the electricity mains through a common regulating rheostat and a voltmeter is bridged across the lamps. Illuminate the lamps and adjust the voltage across them to the value marked on the lamp under test. Move the screen to such position on the bench between the lamps that will yield equal contrasts between the two trapezoidal-shaped figures and their

surrounding semicircular fields. Be sure that the telescope is properly focused on the trapezoids before making settings of the screen. Note the position of the screen carriage. To avoid errors due to dissymmetry in the photometer head, reverse the head, readjust for equality of contrast and record the position of the screen.

Then move the arm carrying the test lamp 20 degrees from its former position and take readings of the screen positions. Repeat this procedure at intervals of 20 degrees until the lamp again reaches its initial position. Arrange all observations in tabular form. Turn out the standard lamp immediately after all readings have been taken.

CONCLUSIONS. The mean reading I_s for the two settings of the screen made at each angular position of the test lamp is obtained by taking the square root of the product of the two observations, but if the difference of the settings is small, the arithmetical mean of the observations is sufficiently accurate. Determine the candle-power I_s of the standardized lamp at the voltage employed from its calibration curve. Compute the candle-power of the test lamp for each position and plot the results on polar coördinate paper. Sketch the position of the lamp around the origin and indicate the scale of candle-power along a single diameter. From this distribution curve determine the mean spherical candle-power of the lamp tested.

APPENDIX

TABLES OF PHYSICAL CONSTANTS

$$1^{\circ} = \frac{.017453 \text{ rad}}{.017453 \text{ rad}} \cdot 1.5708 \text{ rad}$$

$$= 1.5708 \text{ rad}$$

$$1.5708 \text{ rad}$$

$$\frac{1}{6} = \frac{1}{6}$$

$$1.017453$$

$$1.5708 \text{ rad}$$

$$1.5708 \text{ rad}$$

TABLE I
CONSTANTS AND CONVERSION FACTORS

$\pi = 3.1416$	$\epsilon = 2.7183$
$\pi^2 = 9.8696$	$\log_e 10 = \frac{1}{\log_{10} e} = 2.3026$
$\frac{1}{\pi} = 0.31831$	1 radian = 57.2958 degrees
$\log_{10} \pi = 0.49715$	1 degree = 0.017453 radian
1 micron = $\mu = 0.001$ mm.	1 sq. inch = 6.4516 sq. cm.
1 centimeter = 0.39370 inch	1 sq. foot = 929.03 sq. cm.
1 inch = 2.5400 cm.	1 sq. meter = 10.764 sq. feet
1 foot = 30.480 cm.	1 cu. inch = 16.387 cu. cm.
1 meter = 3.2808 feet	1 liter = 1000 cu. cm.
1 kilometer = 0.62137 mile	1 gallon = 3.7853 liters
1 mile = 1.6094 km.	1 cu. foot = 0.028317 cu. meter
1 gram = 15.432 grains	1 pound = 453.59 grams
1 ounce = 28.350 grams	1 kilogram = 2.2046 pounds
1 hour = 3600 seconds	1 mean solar year = 8766 hours
1 sidereal year = 365 days, 6 hours, 9 minutes, 9.314 seconds	
1 pound (wt.) = 444,790 dynes*	1 foot-pound = 1.3549 joules
1 bar = pressure of 0.00075 mm. Hg.	
1 gram per sq. cm. = 0.01422 pound per sq. inch	
1 pound per sq. inch = 70.307 grams per sq. cm.	
1 atmosphere pressure = 14.697 pounds per sq. inch	
1 joule = 10,000,000 ergs	1 B.T.U. = 252.00 calories
1 calorie = 4.186 joules	1 B.T.U. = 778.07 foot-pounds*
1 calorie = 0.42688 kg-meters*	1 horse power = 746 watts†

* For $g = 980.6$ (45° latitude)./

† For $g = 981.2$ (at London).

TABLE II
ACCELERATION OF GRAVITY

Place	cm./sec. ²	Place	cm./sec. ²
Berlin, Germany	981.276	New York, N. Y.	980.225
Boston, Mass.	980.362	Paris, France.	980.943
Chicago, Ill.	980.274	Philadelphia, Pa.	980.138
Denver, Colo.	979.628	Rome, Italy.	980.316
London, England. . . .	981.194	St. Louis, Mo.	979.993
Madison, Wis.	980.352	San Francisco, Cal. . . .	979.935
New Orleans, La.	979.313	Washington, D. C. . . .	980.050

The value of g at any latitude λ and at any altitude h meters may be computed from the equation:

$$g = 980.616 (1 - 0.00266 \cos 2\lambda - 0.00000020 h).$$

TABLE III
HARDNESS OF METALS
(SHORE SCLEROSCOPE SCALE)

Metal	Annealed	Hammered
Babbitt.....	4-9	
Bismuth (cast).....	9	
Brass (cast).....	7-35	
Brass (drawn).....	10-15	20-45
Copper (cast).....	6	14-20
Gold.....	5	8-9
Iron, pure.....	18	25-30
Iron, gray (cast).....	30-45	
Iron, gray (chilled).....		50-90
Lead (cast).....	2-5	3-7
Phosphor bronze.....	20	35
Platinum.....	10	17
Silver.....	6-7	20-30
Steel, carbon, tool (hardened).....		90-110
Steel, chrome-nickel.....	47	
Steel, chrome-nickel (hardened).....		60-75
Steel, high-speed (hardened).....		70-105
Steel, mild, 0.15% carbon.....	22	30-45
Steel, tool, 1.0% carbon.....	30-35	40-50
Steel, tool, 1.65% carbon.....	35-40	
Steel, vanadium.....	35-45	
Tin, pure (cast).....	8	12
Zinc (cast).....	8	20

These figures, given by the Shore Instrument and Mfg. Co., are subject to variations owing to nature of composition or compression of metals.

TABLE IV
COEFFICIENTS OF ELASTICITY

Substance	Stretch Modulus		Shear Modulus	
	10 ¹¹ dynes per sq. cm.	10 ⁹ pounds per sq. in.	10 ¹¹ dynes per sq. cm.	10 ⁹ pounds per sq. in.
Aluminum.....	7.1- 7.3	10.3-10.6	2.5-3.3	3.6- 4.8
Brass.....	8.4-10.2	12.2-14.8	2.6-3.6	3.8- 5.2
Copper.....	10.3-12.9	15.0-18.7	3.8-4.5	5.5- 6.5
Glass.....	6.0- 8.0	8.7-11.6	2.3-2.7	3.3- 3.9
Iron (cast)....	11.5	16.7	5.1-8.3	7.4-12.0
Iron (drawn)..<	19.0-21.3	27.5-31.0		
Steel.....	17.0-21.3	24.7-31.0	7.3-8.4	10.6-12.2

TABLE V
VISCOSITIES OF LIQUIDS

Liquid.	Temperature ° C.	Coefficient of Viscosity dynes per sq. cm.
Alcohol (ethyl).....	0	0.0177
Alcohol (ethyl).....	20	0.0119
Ether.....	10	0.0026
Glycerine.....	2.8	42.20
Glycerine.....	20.3	8.30
Mercury.....	20	0.0157
Olive oil.....	15	0.989
Petroleum.....	17.5	0.0190
Water.....	0	0.0179

SPECIFIC VISCOSITY OF WATER

Temper- ature ° C.	Viscosity	Temper- ature ° C.	Viscosity	Temper- ature ° C.	Viscosity
0	1.000	35	0.404	70	0.226
5	0.849	✓ 40	367	75	212
10	730	45	335	80	199
15	637	50	307	85	187
✓ 20	561	55	283	90	176
25	498	60	262	95	167
30	446	65	243	100	158

TABLE VI
DENSITY AND COEFFICIENTS OF EXPANSION OF
GASES
(At 76 cm. Hg.)

Gas	Density grams per liter at 0° C.	Pressure Coefficient at constant volume (bet. 0 and 100° C)	Coefficient of Cubical Expansion at constant pres- sure (bet. 0 and 100° C)
Air.....	1.2928	0.003665	0.003671
Carbon dioxide...	1.9768	0.003706	0.003710
Chlorine.....	3.1674	0.003807	0.003833
Hydrogen.....	0.0900	0.003656	0.003661
Nitrogen.....	1.2514	0.003668	0.003671
Oxygen.....	1.4292	0.003668	0.003674

TABLE VII
SPECIFIC HEATS OF GASES

Gas	Specific Heat at constant pressure	Ratio of Specific Heat at constant pressure to that at constant volume
Air.....	0.238	1.403
Carbon dioxide.....	0.203	1.300
Chlorine.....	0.124	1.33
Hydrogen.....	3.405	1.408
Nitrogen.....	0.244	1.414
Oxygen.....	0.218	1.398
Steam (at 100° C.).....	0.421	1.33

TABLE VIII
DENSITY AND SPECIFIC HEATS OF SOLIDS AND LIQUIDS
AT ORDINARY TEMPERATURES

Substance	Density gr./cu.cm.	Specific Heat
Alcohol, ethyl (0° C.).....	0.81	0.55
Aluminum.....	2.6- 2.8	0.212
Antimony.....	6.2- 6.7	0.050
Bismuth.....	9.7- 9.9	0.030
Brass.....	8.2- 8.7	0.088-0.095
Carbon bisulphide (0° C.).....	1.29	0.235
Copper.....	8.4- 8.9	0.093
German silver.....	8.3- 8.8	0.095
Glass.....	2.4- 6.3	0.11-0.20
Glycerine.....	1.27	0.576
Iron (wrought).....	7.8- 7.9	0.115
Lead.....	11.3-11.4	0.031
Marble.....	2.6- 2.8	0.21
Mercury (0° C.).....	13.596	0.0334
Nickel.....	8.6- 8.9	0.113
Platinum.....	21.4	0.032
Silver.....	10.4-10.6	0.056
Steel.....	7.6-7.8	0.117
Sugar.....	1.61	0.304
Tin.....	7.0- 7.3	0.055
Turpentine (10° C.).....	0.87	0.43
Vulcanite.....	1.2	0.331
Water (4° C.).....	1.000	1.003
Zinc.....	6.9- 7.2	0.093

TABLE IX

LOCAL GEOGRAPHICAL DATA—*Brooklyn, N. Y.*

Latitude of Polytechnic Institute.....	40° 41' 33"
Longitude of Polytechnic Institute.....	73° 59' 28" West
Altitude of Physical Laboratory floor above mean sea level	23.386 meters
Value of g in Physical Laboratory.....	980.220 cm/sec ²

TABLE X

HEATS OF FUSION AND VAPORIZATION

Substance	Melting- point ° C.	Heat of Fusion cal./gram	Boiling- point ° C.	Heat of Vaporization cal./gram
Alcohol (ethyl)....	-130		78.4	205
Benzol.....	5.6	30.6	80.2	93.5
Bismuth.....	270	12.6	1430	
Bromine.....	-7.3	16.2	61	45.6
Carbon bisulphide.	-110		46.2	83.8
Ether (ethyl)....	-118		34.6	90
Lead.....	326	5.4	1525	
Mercury.....	-38.7	2.8	357	65
Paraffin.....	52	35.1	390	
Platinum.....	1755	27.2		
Sulphur.....	115	9.4	444.7	
Water.....	0	79.9	100	535.9
Zinc.....	419	28.1	930	

TABLE XI

HEATS OF COMBUSTION

Substance	Calories per gram	Substance	Calories per gram
Acetylene.....	11900	Hydrogen.....	34000
Alcohol (ethyl)....	7200	Illuminating gas..	5200-6000
Coal (American)...	6500-8300	Petroleum.....	11000
Coal gas.....	5800-11000	Sulphur.....	2200
Dynamite (75%)..	1290	Woods (various) ..	4000-4900

227

TABLE XII

PRESSURE OF WATER VAPOR (Saturated)
(Inches of Hg.)

Temper- ature ° F.	Vapor Pressure	Temper- ature ° F.	Vapor Pressure	Temper- ature ° F.	Vapor Pressure
0	0.0383	34	0.195	68	0.684
1	403	35	203	69	707
2	423	36	211	70	732
3	444	37	219	71	757
4	467	38	228	72	783
5	491	39	237	73	810
6	515	40	247	74	838
7	542	41	256	75	866
8	570	42	266	76	896
9	600	43	277	77	926
10	631	44	287	78	957
11	665	45	298	79	989
12	699	46	310	80	1.022
13	735	47	322	81	056
14	772	48	334	82	091
15	810	49	347	83	127
16	850	50	360	84	163
17	891	51	373	85	201
18	933	52	387	86	241
19	979	53	402	87	281
20	0.1026	54	417	88	322
21	108	55	432	89	364
22	113	56	448	90	408
23	118	57	465	91	453
24	124	58	482	92	499
25	130	59	499	93	546
26	136	60	517	94	595
27	143	61	536	95	645
28	150	62	555	96	696
29	157	63	575	97	749
30	164	64	595	98	803
31	172	65	616	99	859
32	180	66	638	100	916
33	187	67	661	101	975

TABLE XIII
THERMAL CONDUCTIVITY

Substance	Temperature °C.	Conductivity calories per cm. per sec. per °C.
Air.....	0	0.000046-0.000057
Aluminum.....	18	0.48
Copper (pure).....	20	0.89-0.95
Flannel.....	50	0.000036
Glass.....	15	0.0011-0.0023
Iron (wrought).....	20	0.15-0.21
Marble.....	30	0.0060
Mercury.....	50	0.018
Silver.....	18	1.006
Water.....	12	0.0013

TABLE XIV
INDICES OF REFRACTION

Solar Line Substance Wave-length (μ) Color	<i>B</i> <i>O</i>	<i>C</i> <i>H</i>	<i>D</i> <i>Na</i>	<i>E</i> <i>Ca</i>	<i>F</i> <i>H</i>	<i>G</i> <i>Fe, Ca</i>	<i>H</i> <i>H, Ca</i>
	0.6870 red	0.6563 red	0.5893 yellow	0.5270 green	0.4862 blue	0.4308 violet	0.3969 violet
Alcohol (15° C.).....	1.3599	1.3606	1.3624	1.3647	1.3667	1.3705	1.3736
Carbon bisulphide (20° C.).....	1.6150	1.6185	1.6277	1.6407	1.6526	1.6767	1.7001
Crown glass ($\delta = 2.5$).....	1.5118	1.5127	1.5153	1.5186	1.5214	1.5267	1.5312
Flint glass ($\delta = 4.7$).....	1.7406	1.7434	1.7515	1.7623	1.7723	1.7922	1.8110
Water (20° C.).....	1.3309	1.3311	1.3330	1.3352	1.3372	1.3412	1.3435

Accepted primary wave-length standard is that of the red cadmium line in air (76 cm. Hg. and 15° C.): 0.000064384696 cm.

TABLE XV
CONSTANTS OF THE NORMAL HUMAN EYE

Part of Eye	Radii of Curvature cm.		Thickness cm.	Index of Re- fraction
	Front Surface	Rear Surface		
Cornea.....	+0.783	-0.733	0.05-0.06	1.351
Aqueous humor.....	+0.733	-1.00 (-0.60)	0.36 (0.32)	1.337
Crystalline lens.....	+1.00 (+0.60)	+0.60 (+0.55)	0.36 (0.40)	1.437
Vitreous humor.....	-0.60 (-0.55)	+1.2	1.59	1.337

The values in the parentheses are applicable to the accommodated eye.

TABLE XVI
VELOCITY OF SOUND

Substance	Temperature °C.	Velocity meters per sec.
Air.....	0	331.7
Alcohol.....	12.5	1241
Aluminum.....	15	5100
Brass.....	15	3500
Copper.....	20	3560
Glass.....	15	5000-6000
Hydrogen.....	0	1286.4
Illuminating gas.....	0	490.4
Iron.....	20	5130
Oxygen.....	0	317.2
Water.....	13	1441
Woods (various).....		1000-4700

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$$-\frac{2.3026}{3.68} \times -0.09151 = \frac{+0.057}{.057}$$

$$\begin{array}{r} -0.057 \\ 105 \overline{) 11.07125} \\ \underline{1125} \\ 2720 \\ \underline{2535} \end{array}$$



$$-\frac{2.3026}{T} \times \log_{10} \delta$$

$$\delta = .81$$

$$T = 3.65 \text{ sec}$$

$$\log_{10} \delta = 9.0849 - 10$$

$$\delta = \epsilon^{-ft} = .09151$$

$$s = \frac{1}{2} at^2$$

$$v = \sqrt{2as}$$

$$v = at$$

$$at = \sqrt{2as}$$

$$at^2 = 2as$$

$$a = \frac{2s}{t^2}$$

$$\left(a = \frac{2s}{t^2} \right)$$

$$\begin{array}{r}
 2.3026 \\
 .09151 \\
 \hline
 23026 \\
 115130 \\
 23026 \\
 207234
 \end{array}$$

83
80
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84
83
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81

$$210710926$$

$$\frac{mg}{m}$$

$$14 + m$$

$$\frac{a(M + m)}{m} = g$$

$$(74^2)$$

82

83
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$$(183)$$

87

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$$9(757)$$

84

$$40 \text{ sp} / 10^\circ$$

$$1^\circ = 4 \text{ sp.}$$

460

139

110

87 =

71

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80

$$91723$$

81.0

$$1 \text{ in} = 200 \text{ div.}$$

1 div.

$$\frac{1}{200} \text{ mm.}$$

$$\begin{array}{r} 2.54 \\ \cdot 519 \\ \hline 20763 \\ 254 \\ \hline 1270 \\ \hline 1.31616 \end{array}$$

8760. pers.

$$(h + vl) = 1188$$

$$\begin{array}{r} 178 \\ \cdot 31 \\ \hline 178 \\ 534 \\ \hline 5518 \end{array}$$

$$\begin{array}{r} 178.4 \\ 178. \overline{) 3100.0} \\ \underline{178} \\ 1320 \\ \underline{1246} \\ 740 \\ \underline{712} \\ 28 \end{array}$$

242.30

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21

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1000

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4.41

1089

4.3021

980

1960

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455.8

4.3

19600.0

172

240

250

218

350

390

